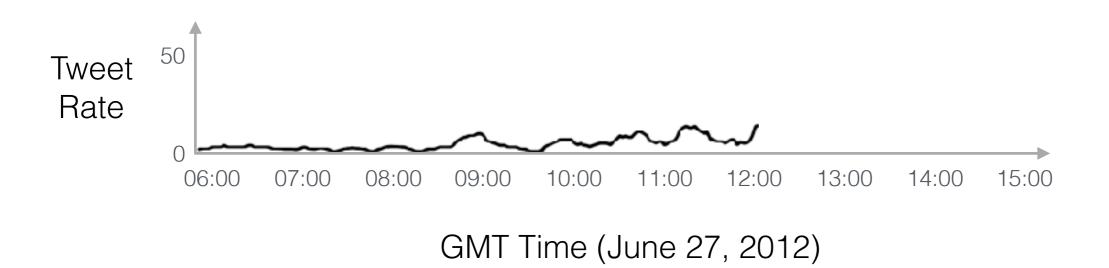
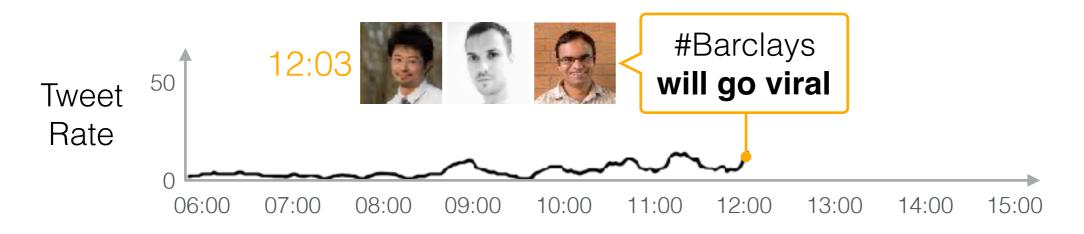
Adaptive Nearest Neighbor Classification and Regression Based on Decision Trees

slides by
George Chen
Carnegie Mellon University
Fall 2017

NN and Kernel Classification and Regression







GMT Time (June 27, 2012)

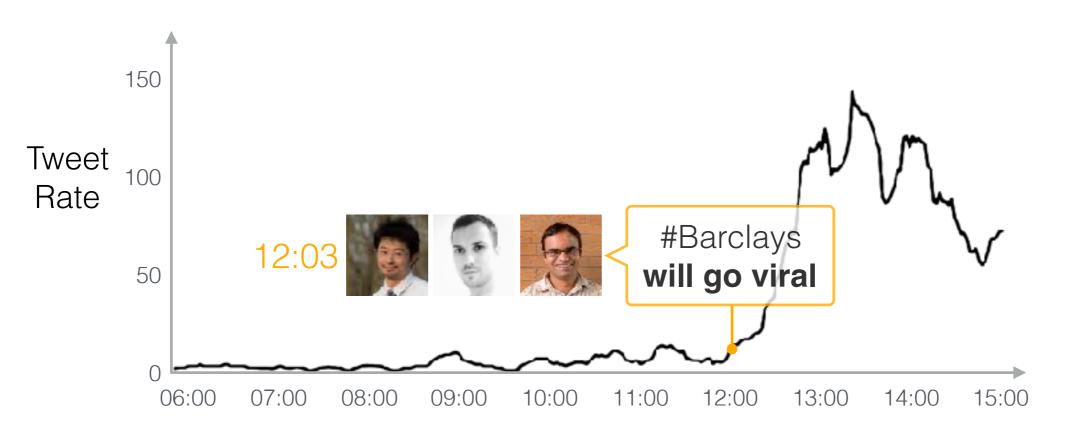
Tweet Rate



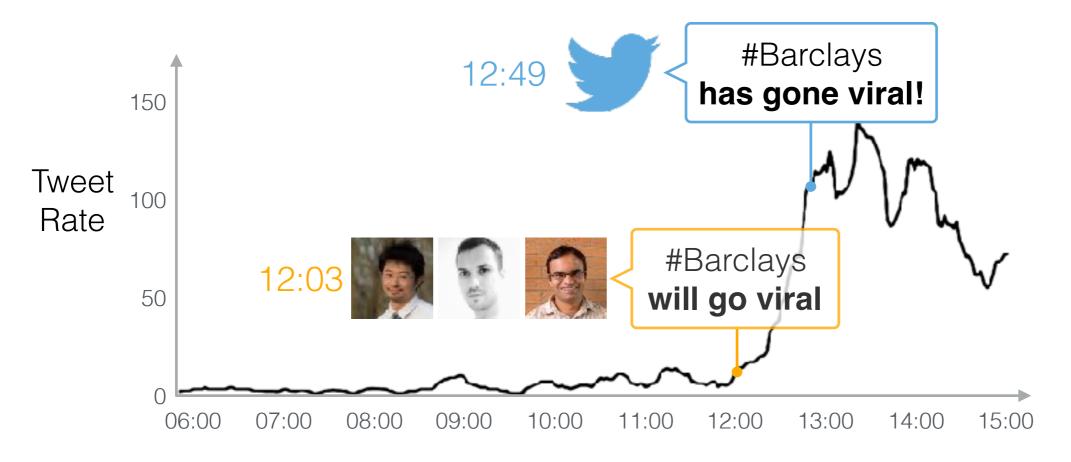
GMT Time (June 27, 2012)



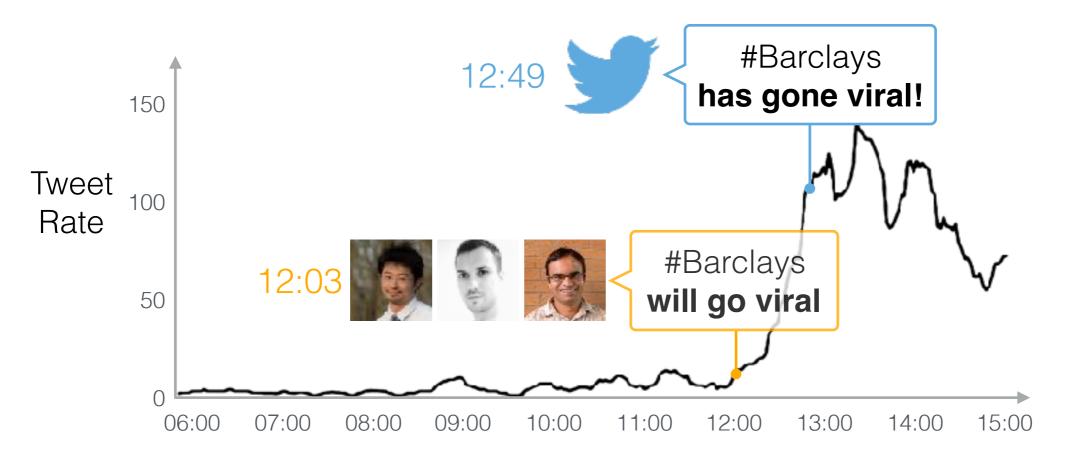
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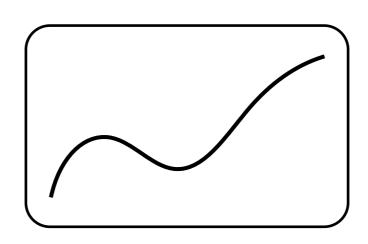
GMT Time (June 27, 2012)

How we did this: weighted majority voting

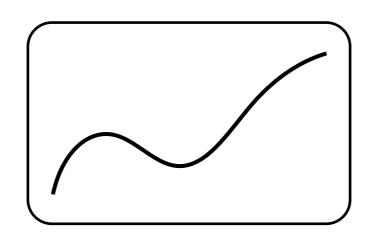
Chen, Nikolov, and Shah. A Latent Source Model for Nonparametric Time Series Classification.

NIPS 2013.

Test data

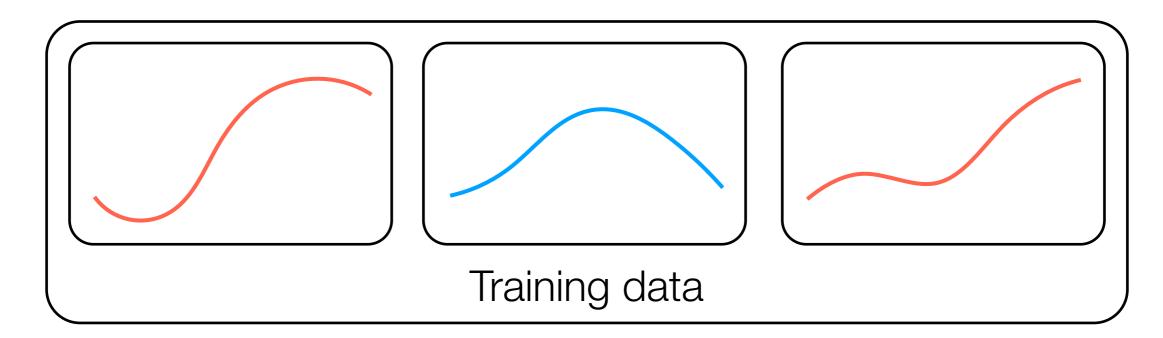


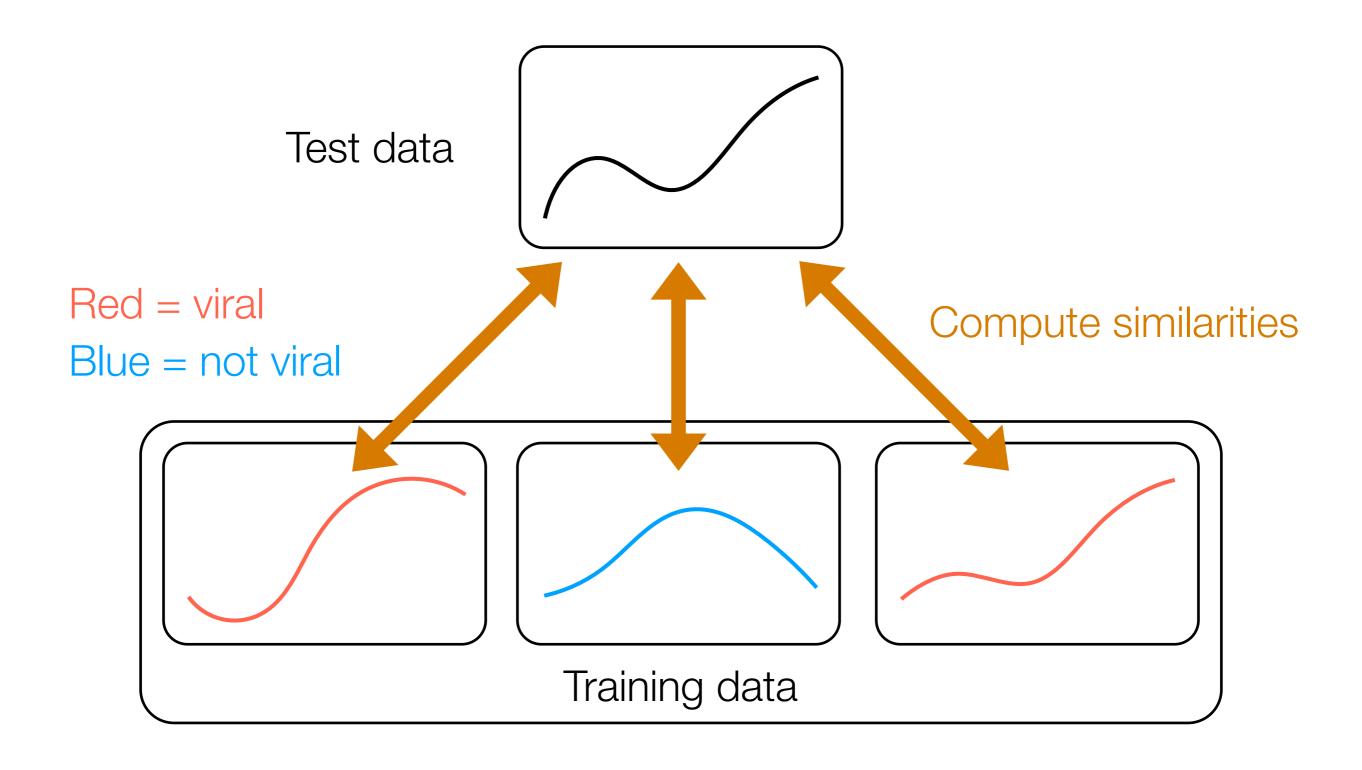
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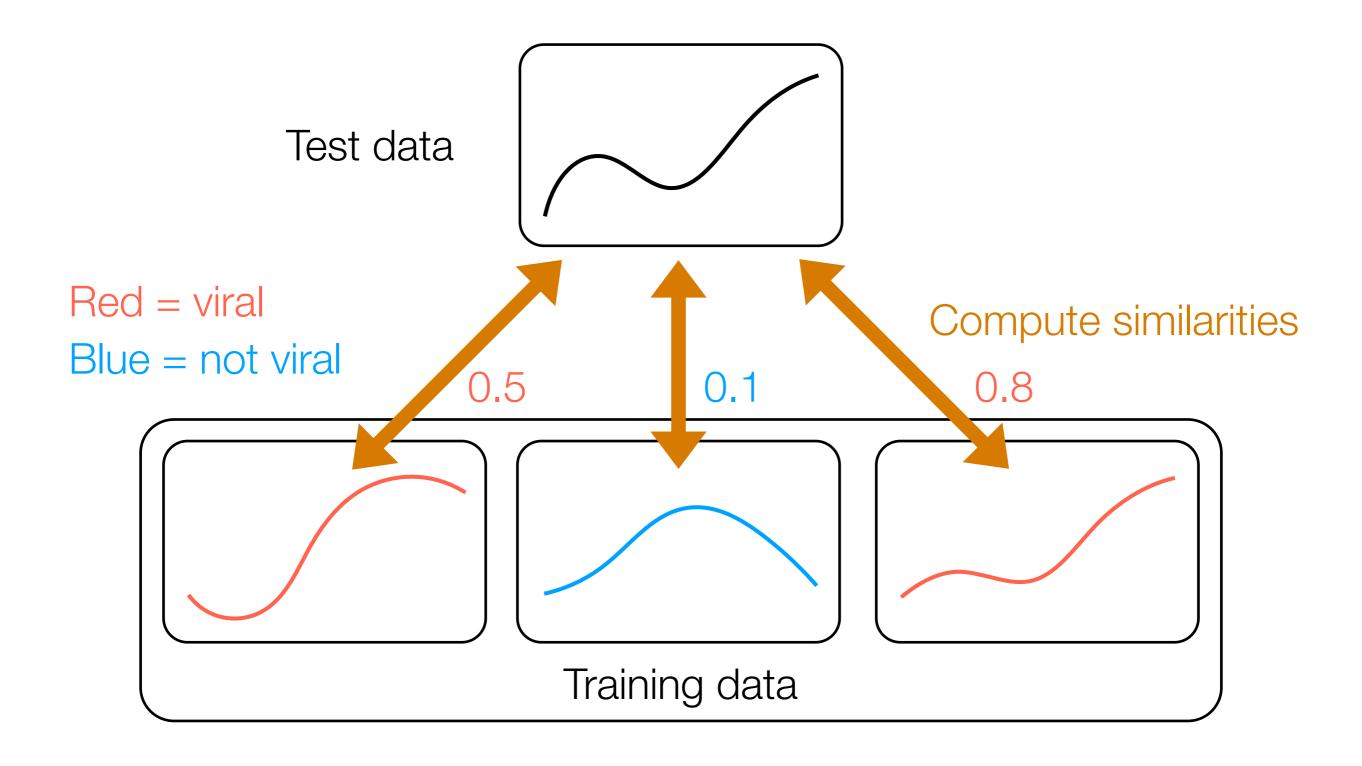


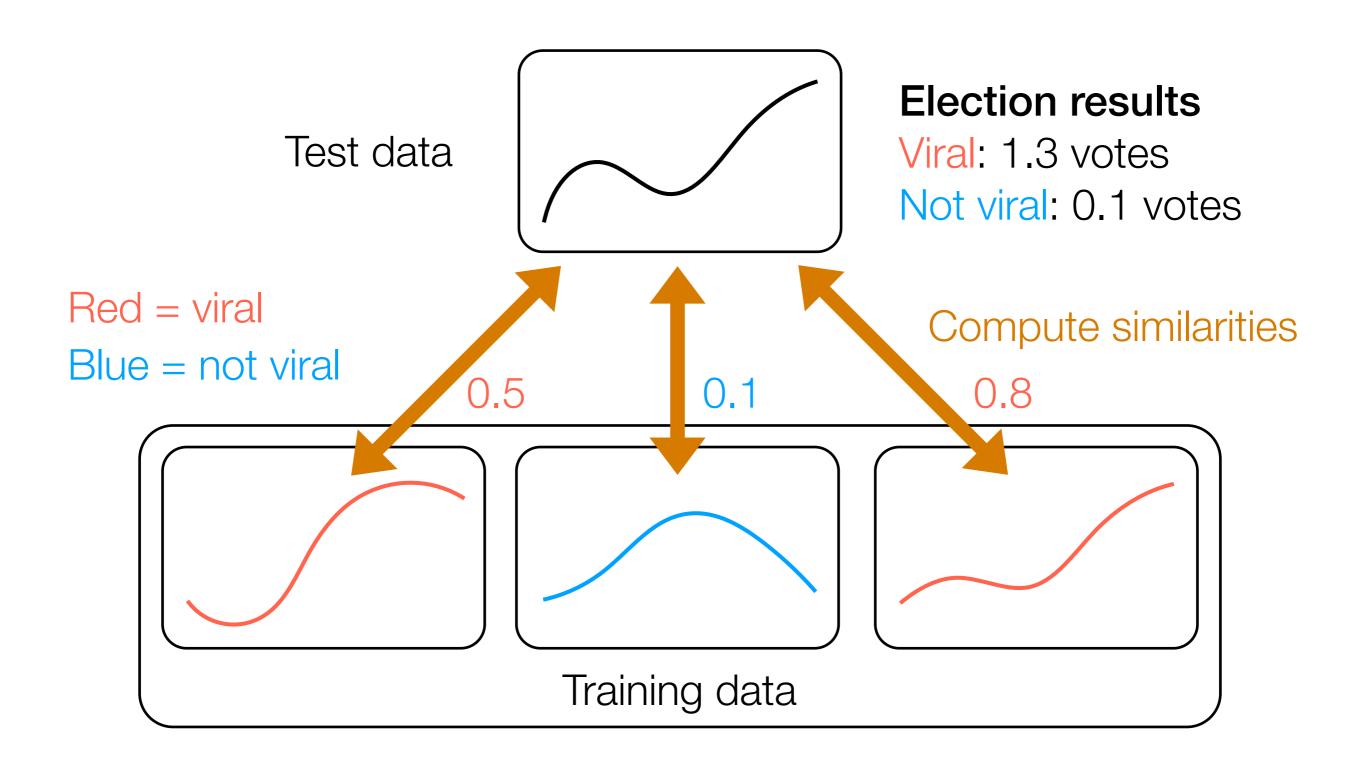
Red = viral

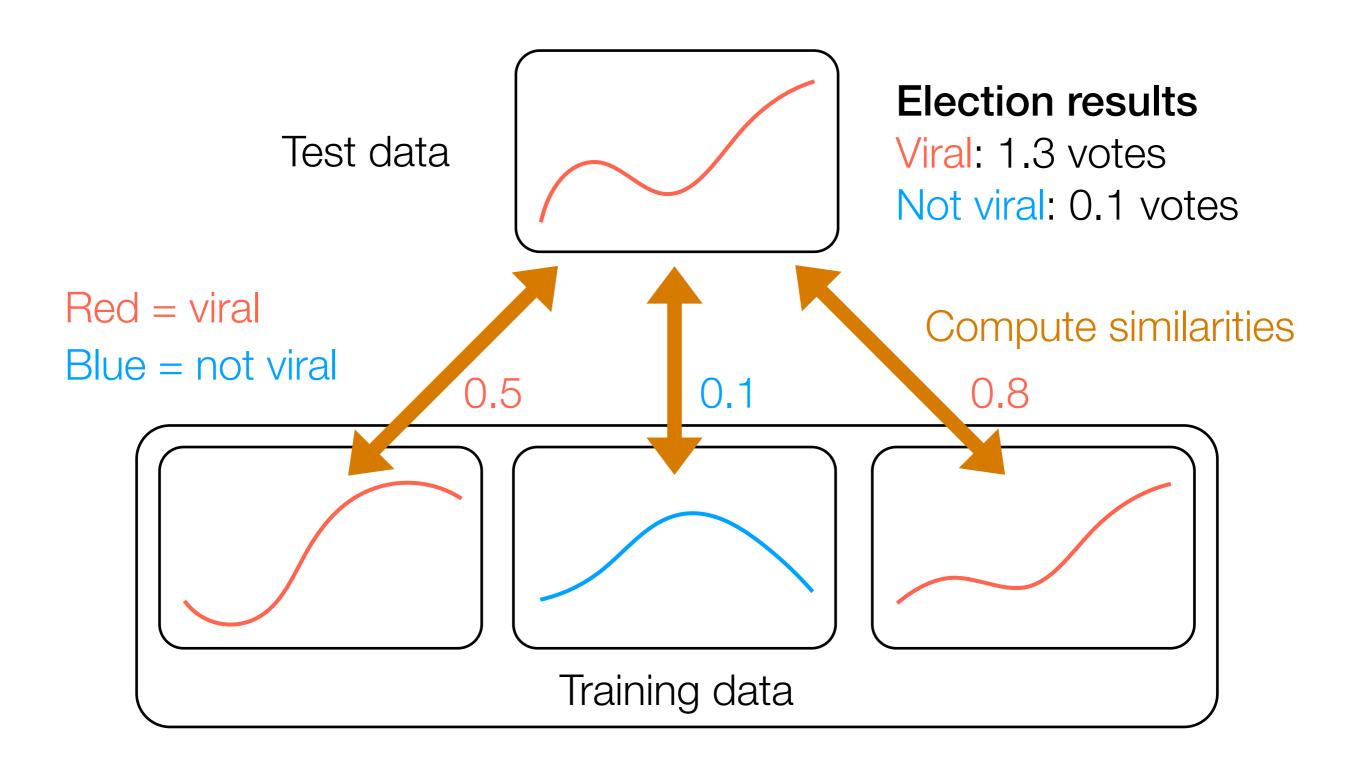
Blue = not viral

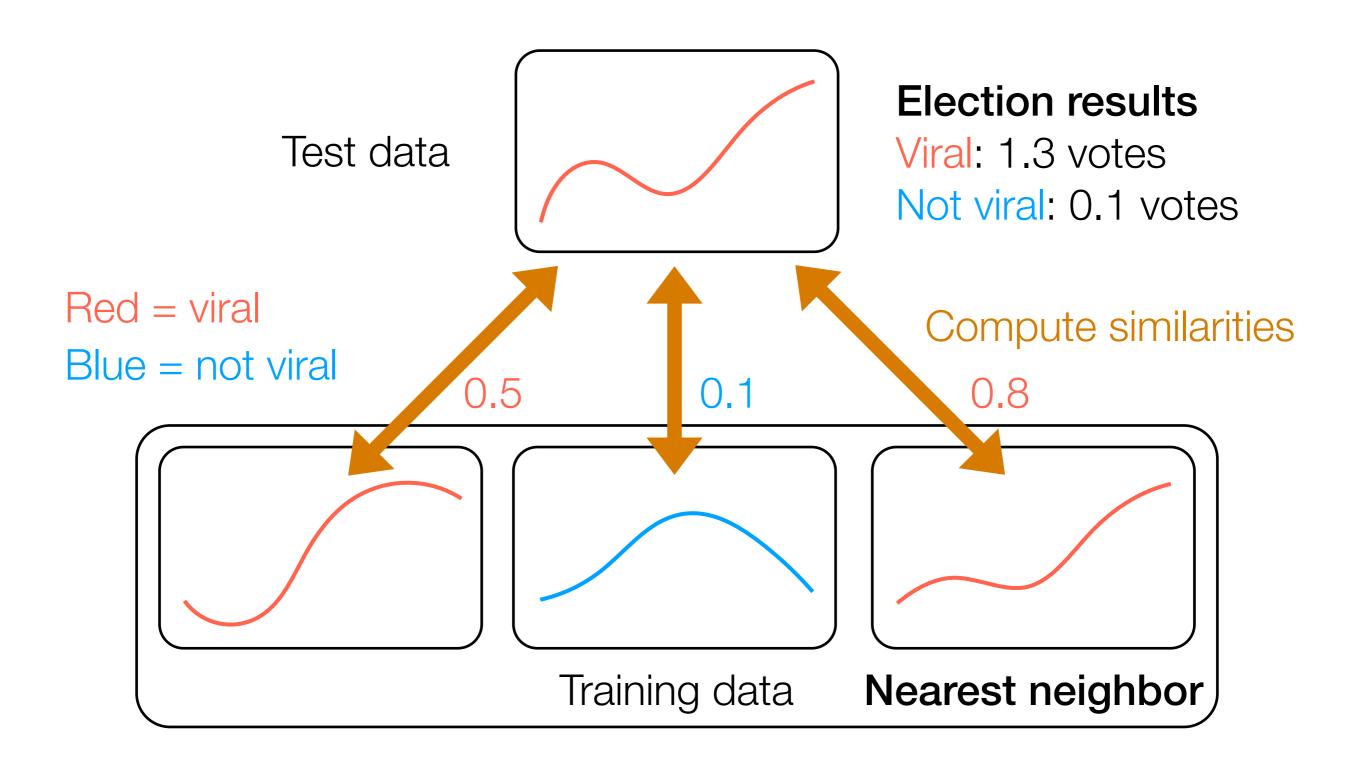


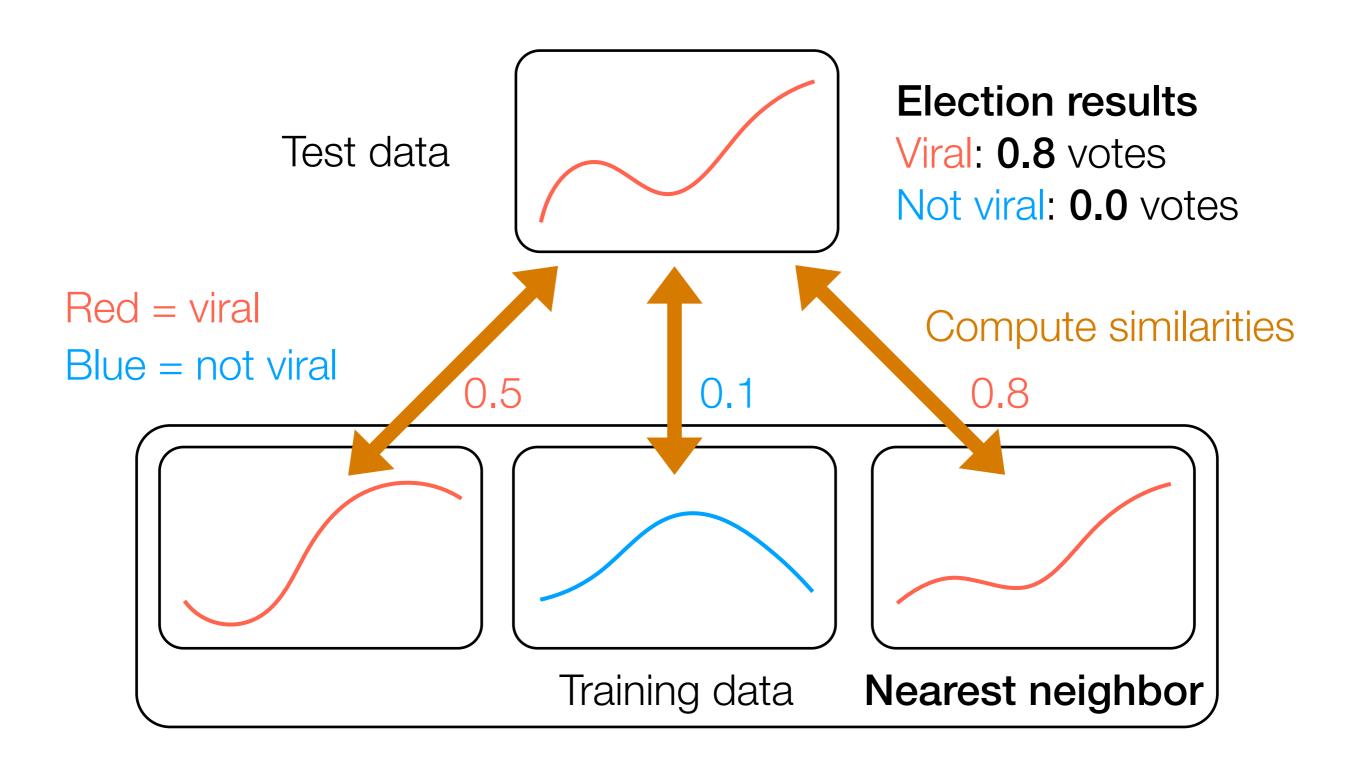




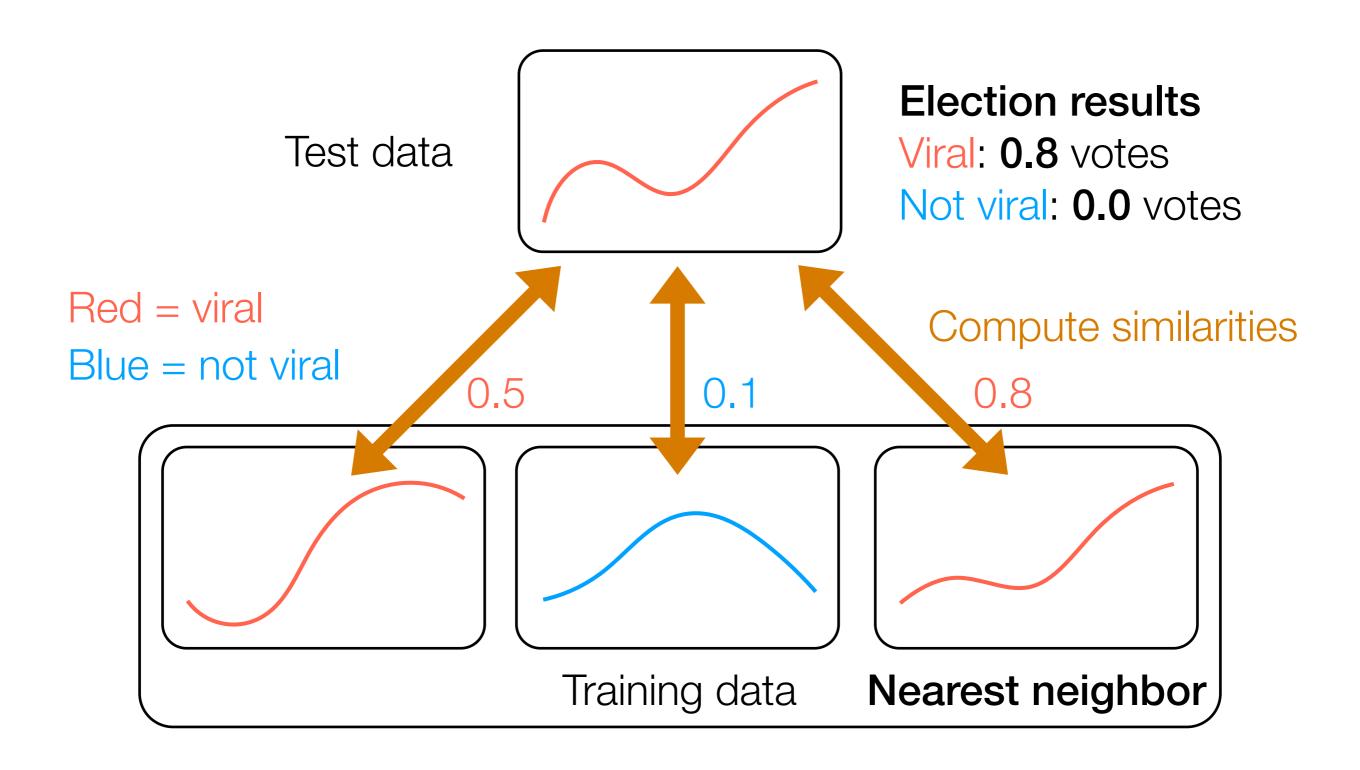








Nearest Neighbor Classification



• **k-NN classification:** consider **k** most similar training data to test data point

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 - Weighted: when tallying up votes, use the similarities that we computed

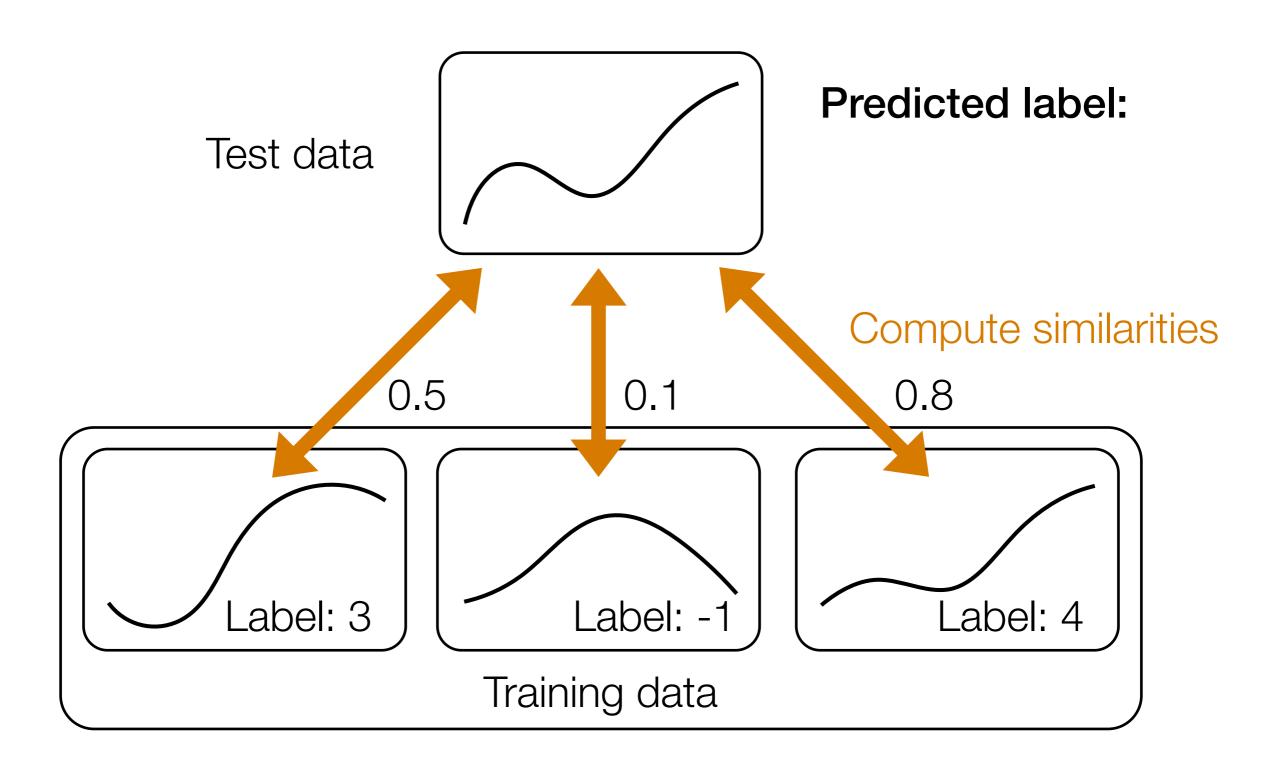
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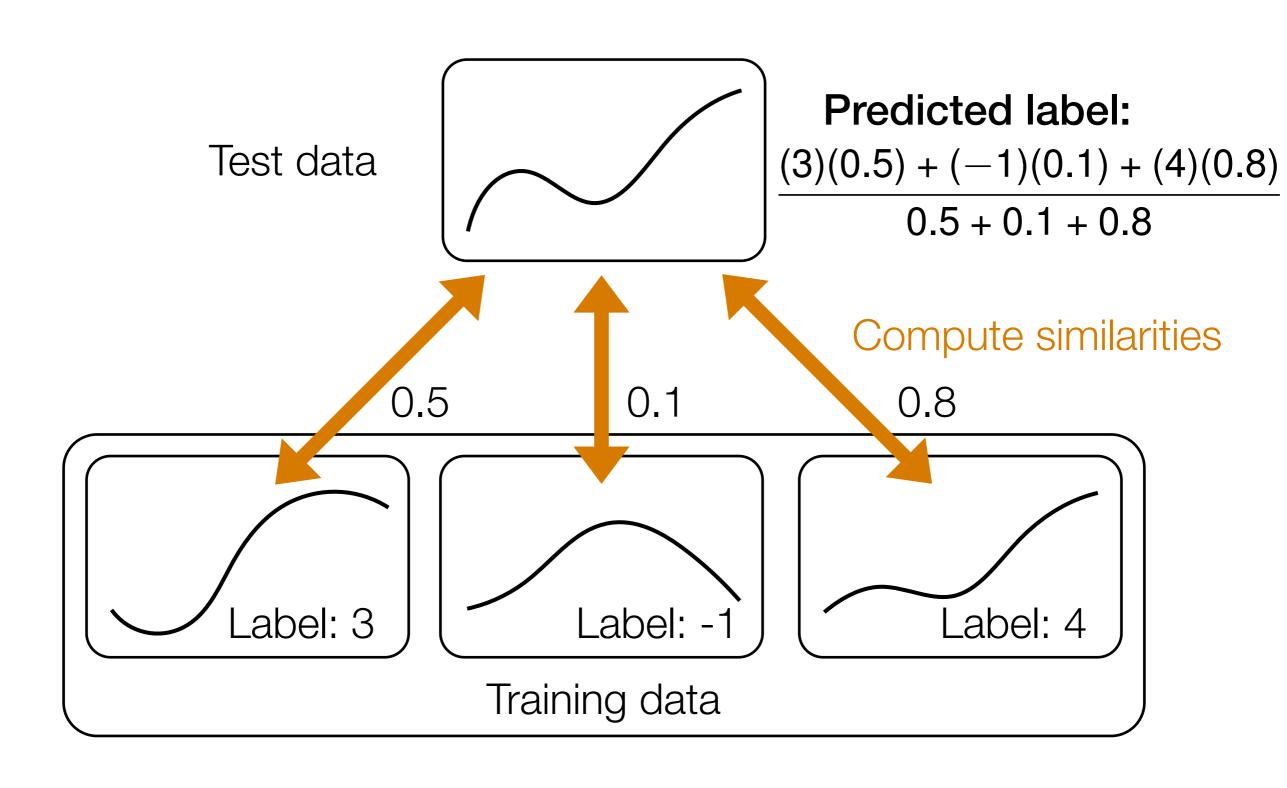
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 - Once again, can use weighted or unweighted votes

Regression: Each label is continuous instead of discrete

Kernel Regression

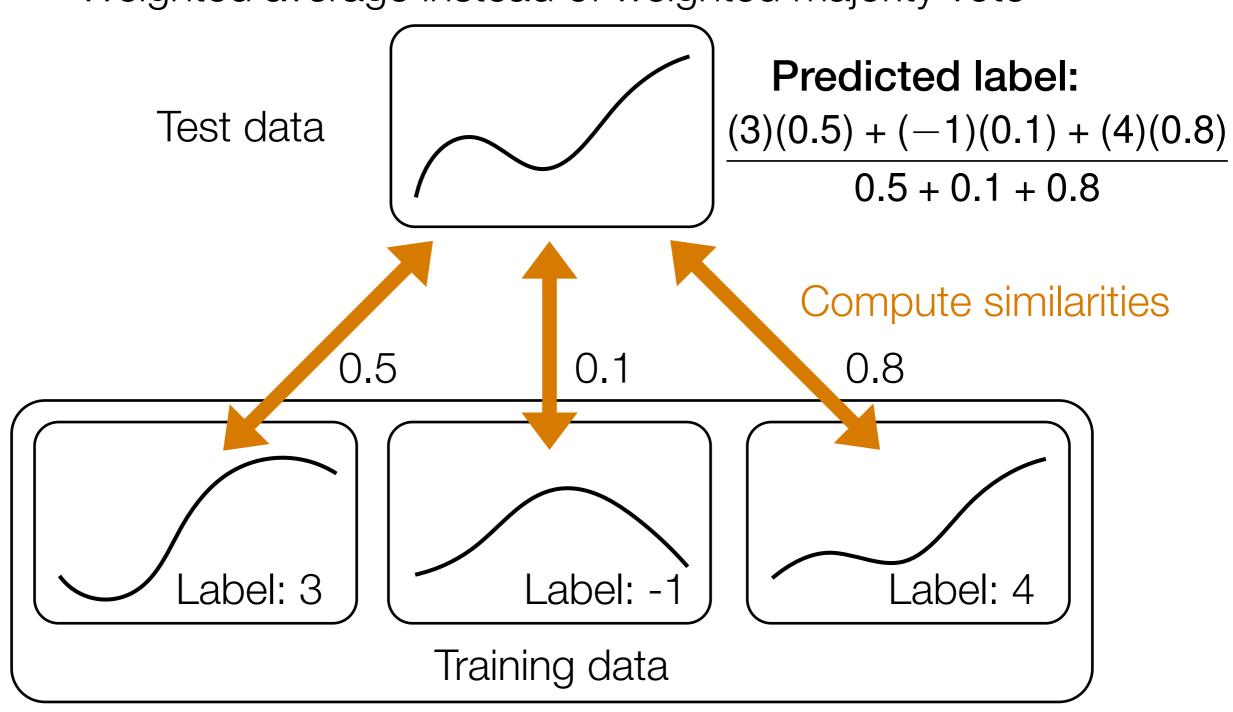


Kernel Regression

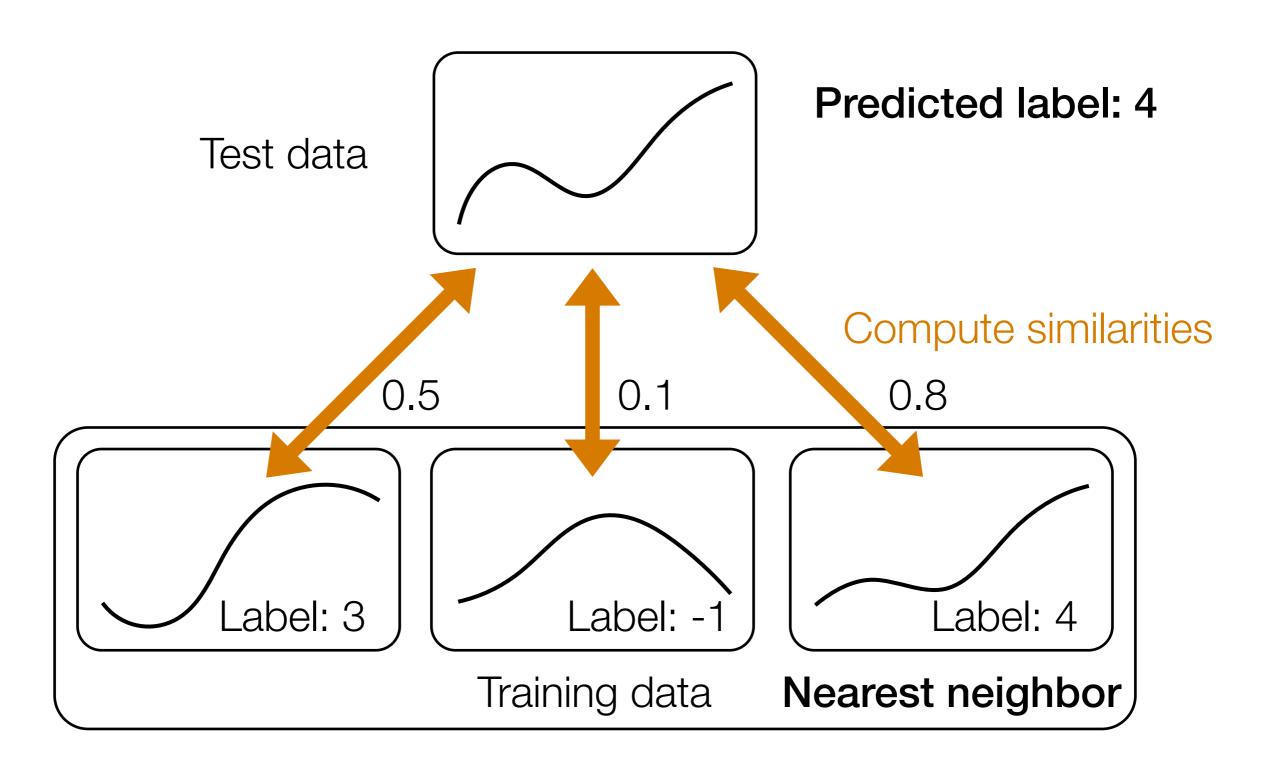


Kernel Regression

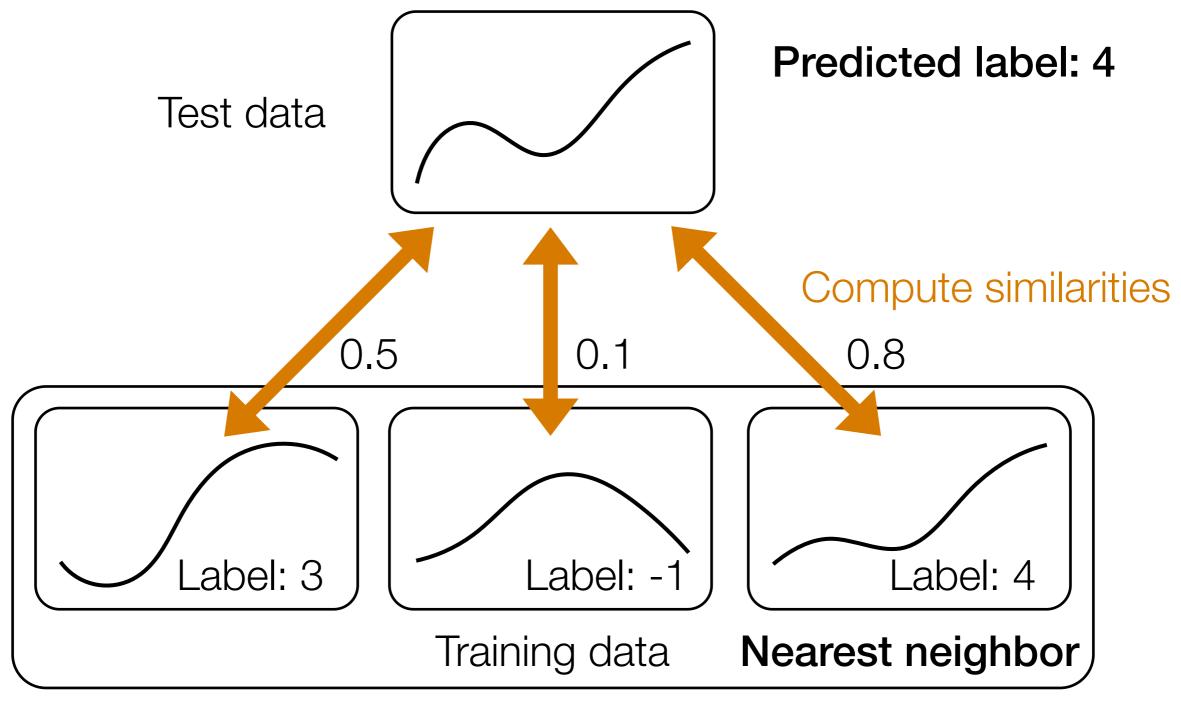
Weighted average instead of weighted majority vote



NN Regression



NN Regression

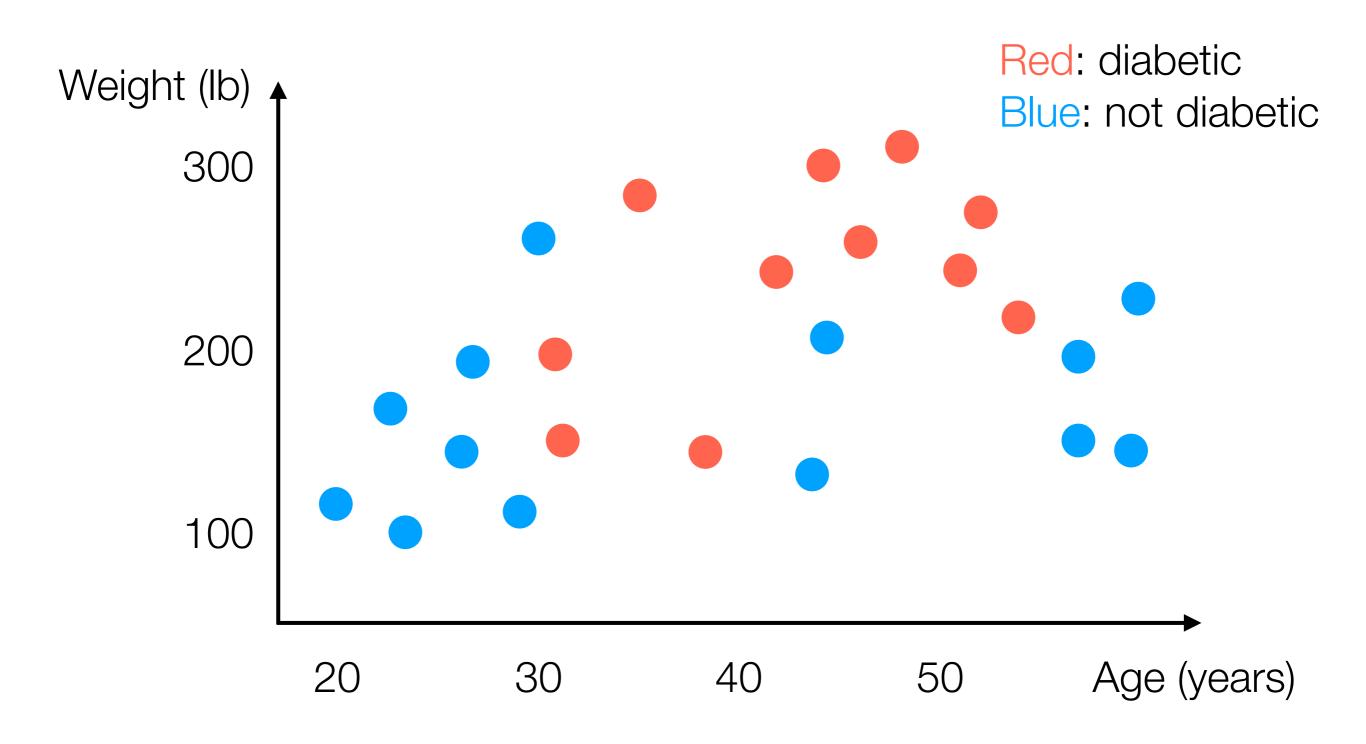


Just like classification: *k*-NN and fixed-radius NN variants, also weighted and unweighted

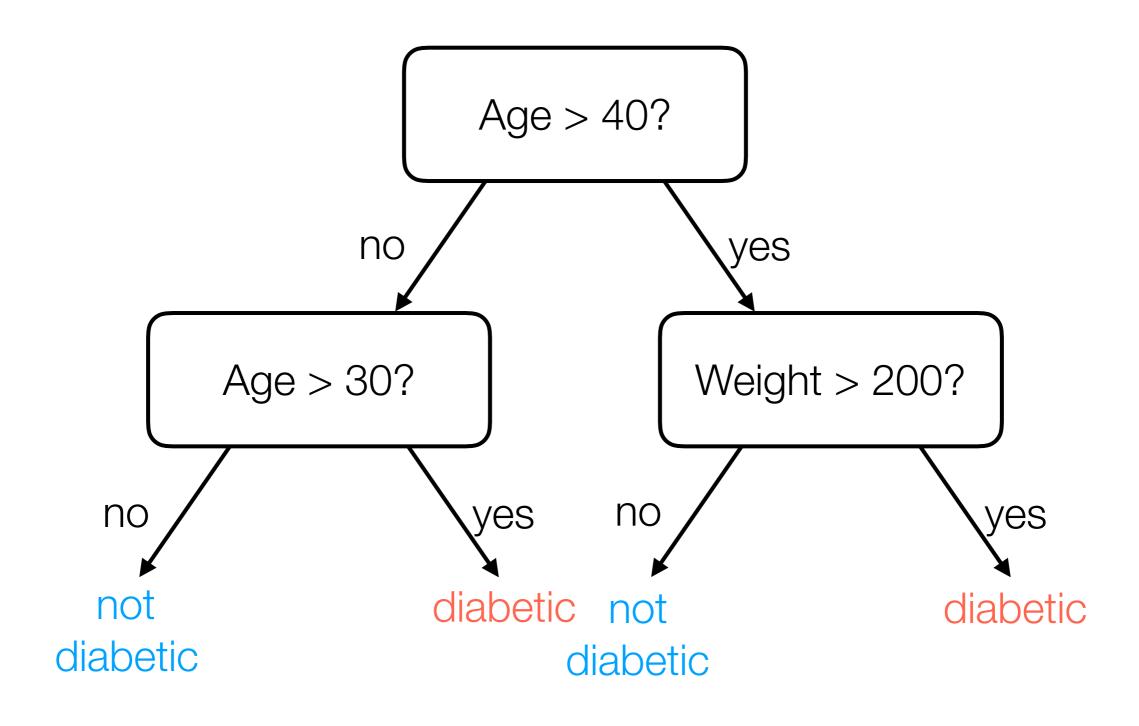
"Adaptive" nearest neighbors: learn the similarity function

Decision Trees

Example Made-Up Data



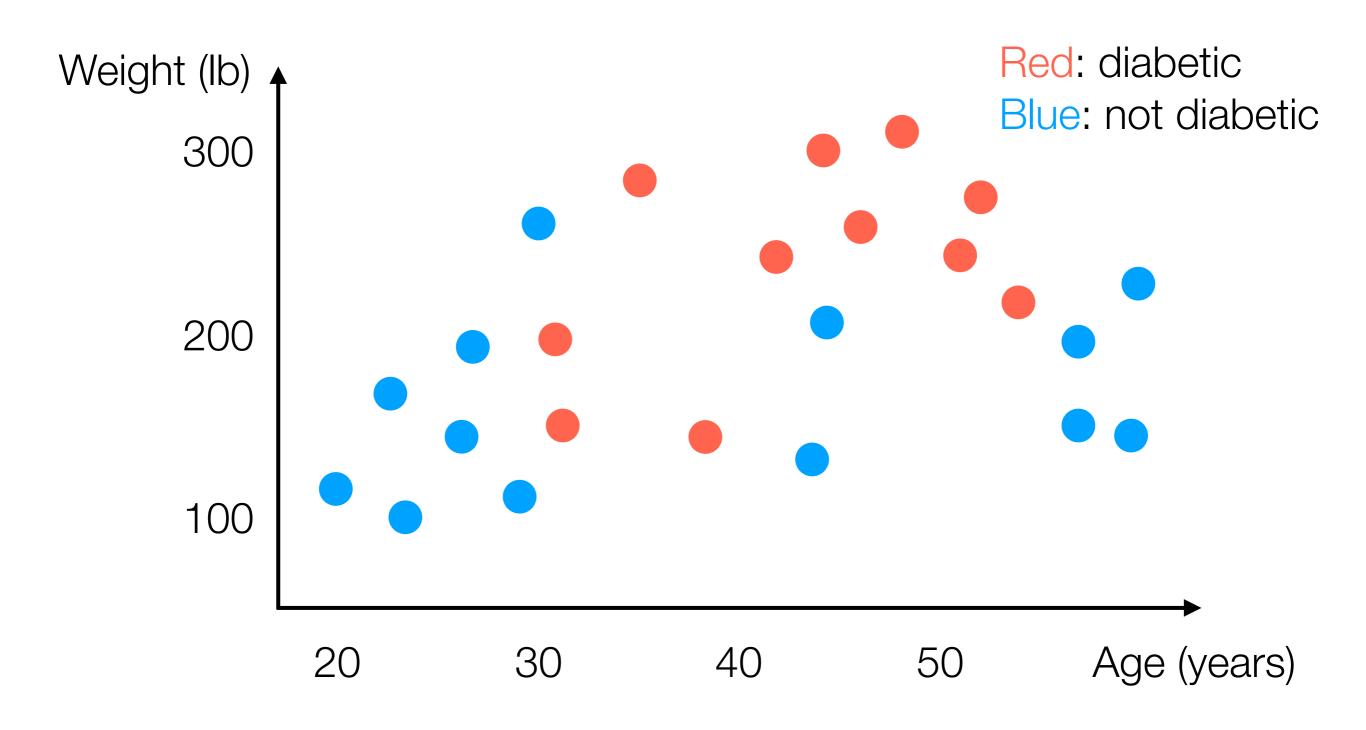
Example Decision Tree

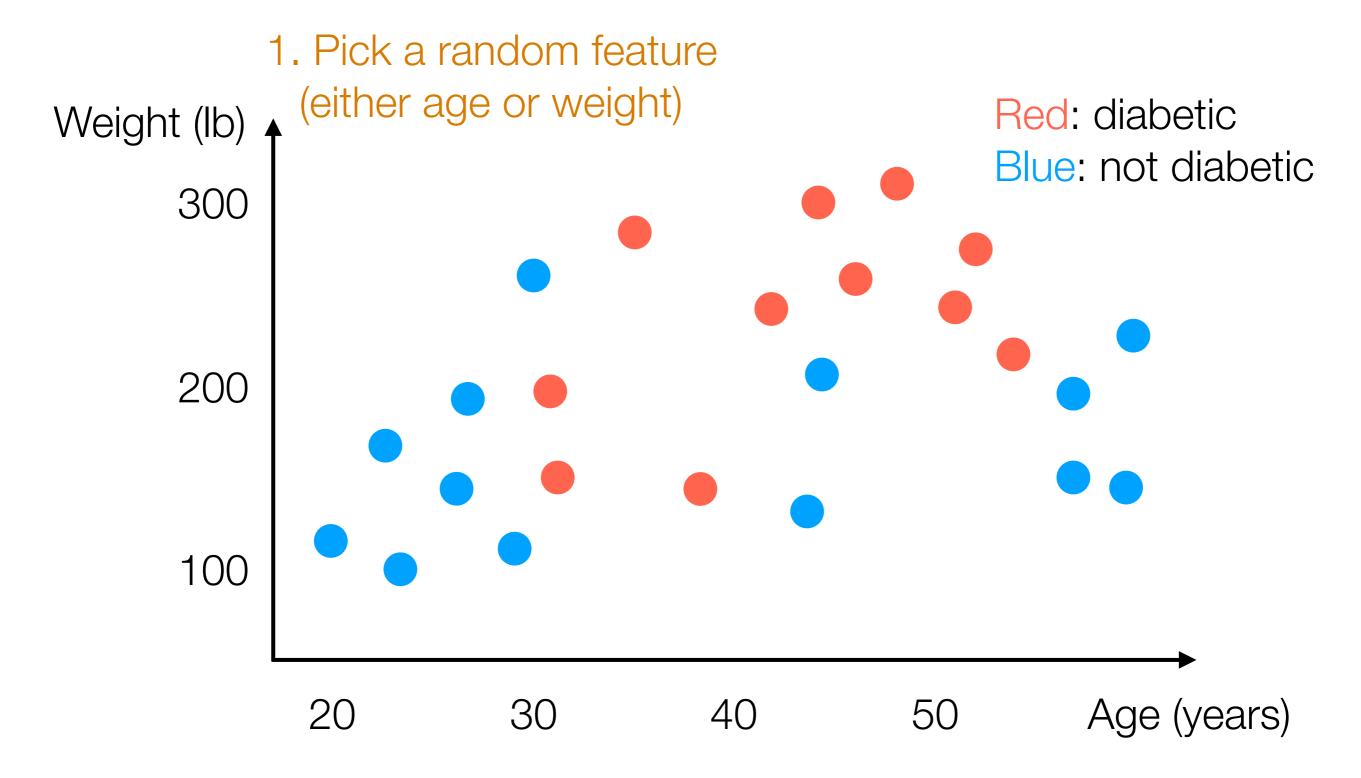


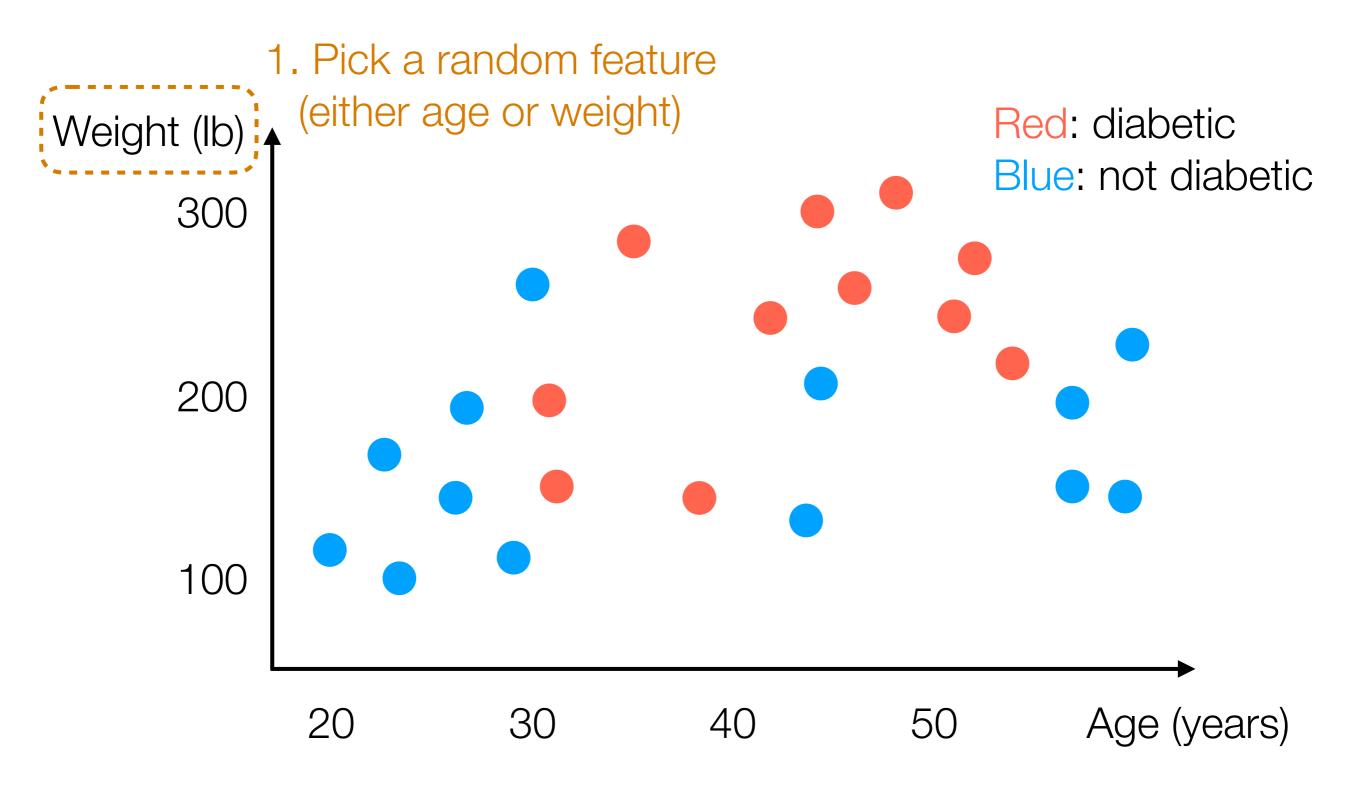
 Many ways: general approach actually looks a lot like divisive clustering but accounts for label information

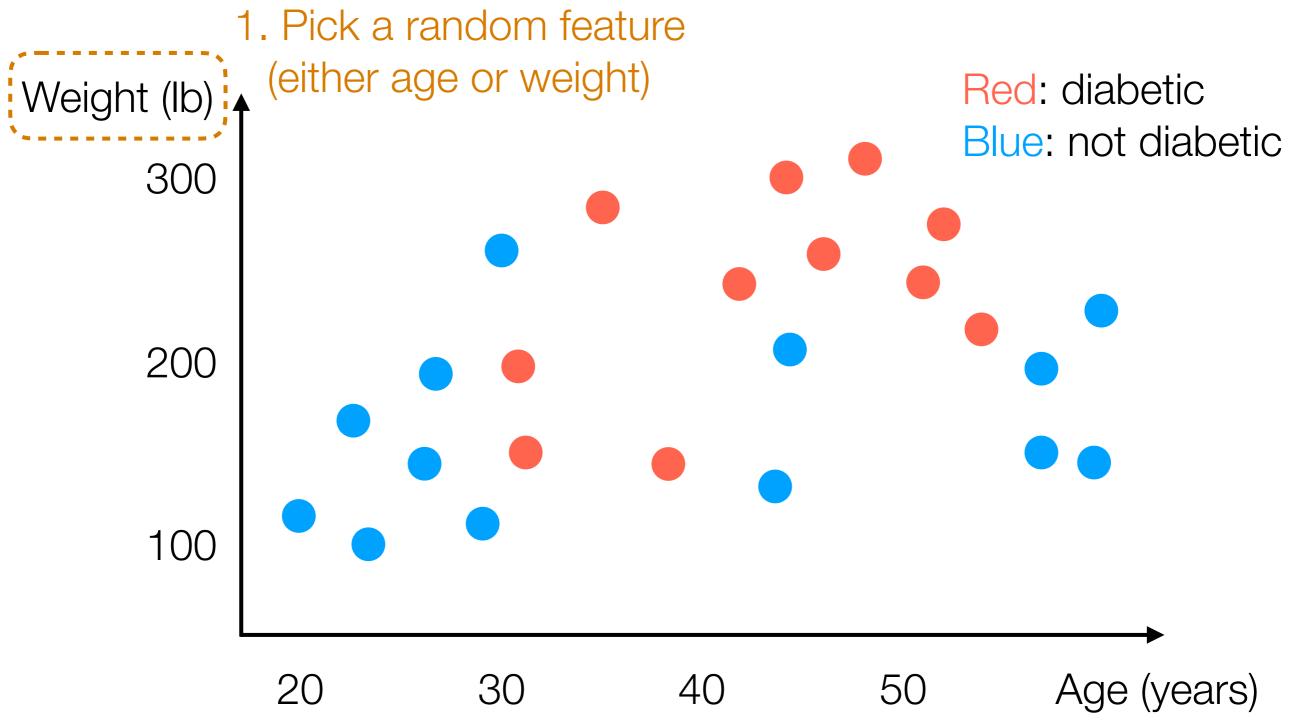
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 I'll show one way (that nobody actually uses in practice) but it's easy to explain

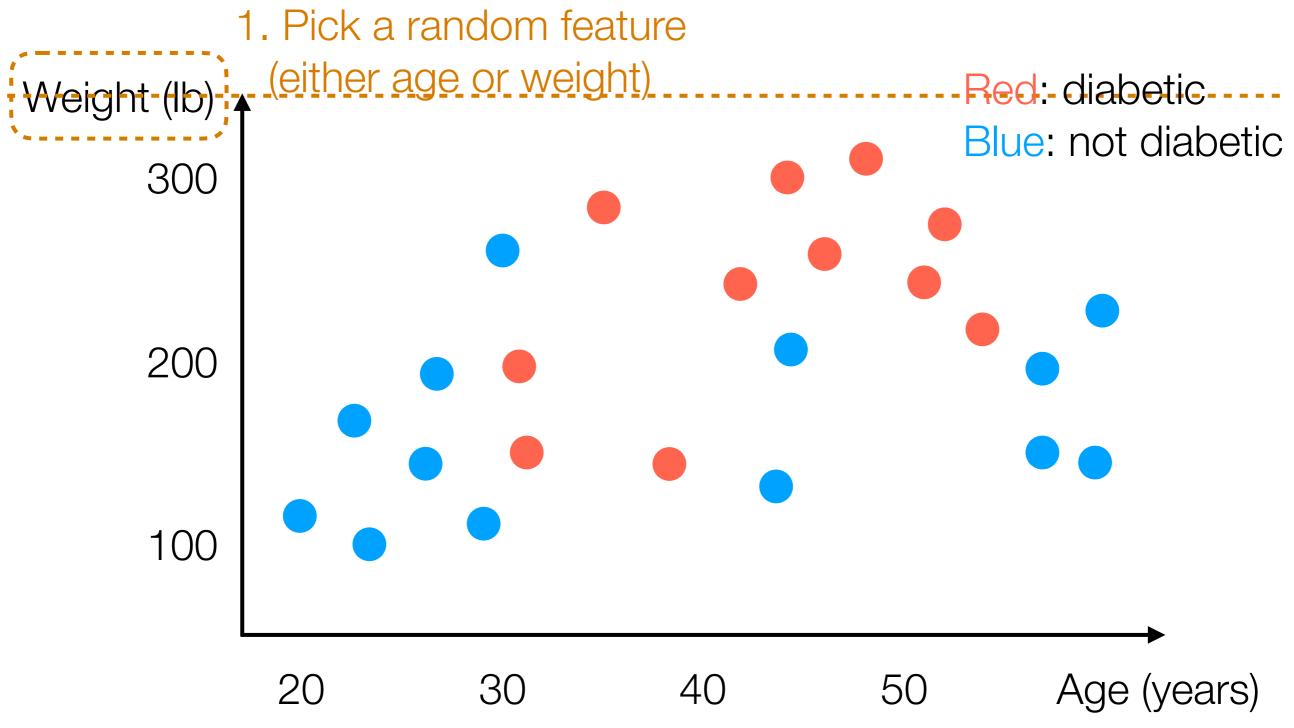




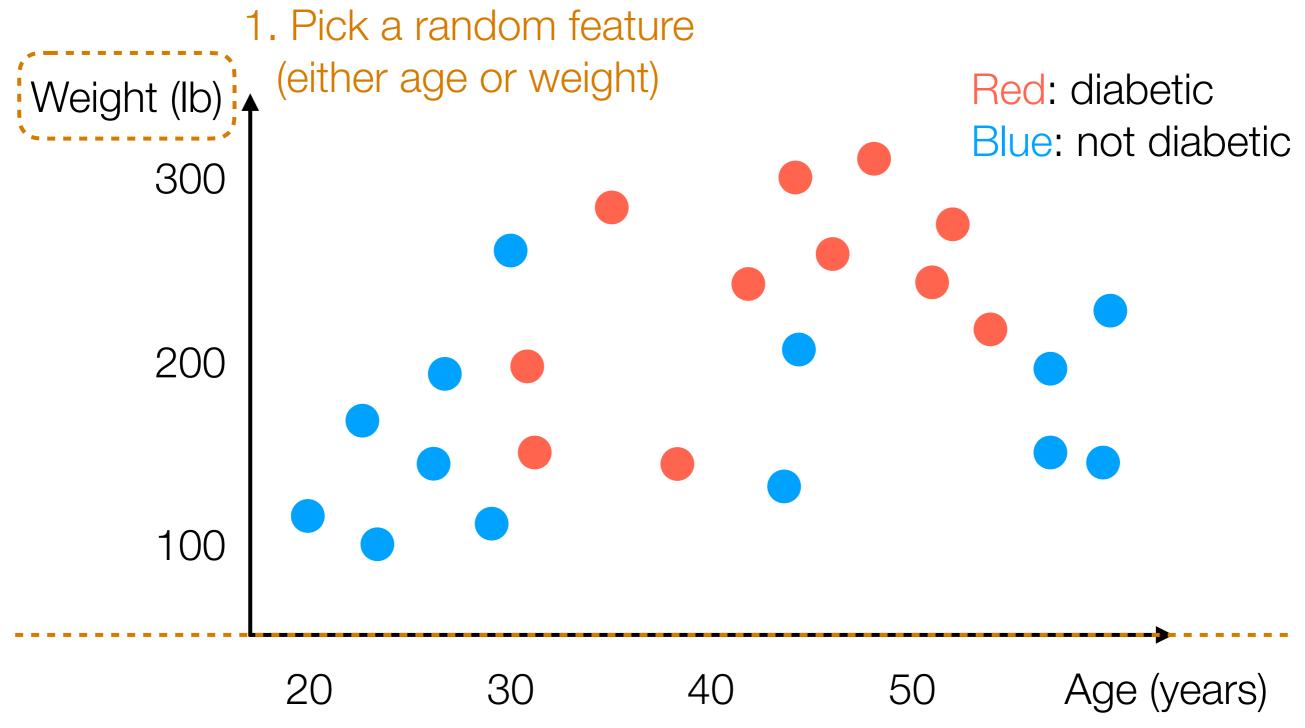




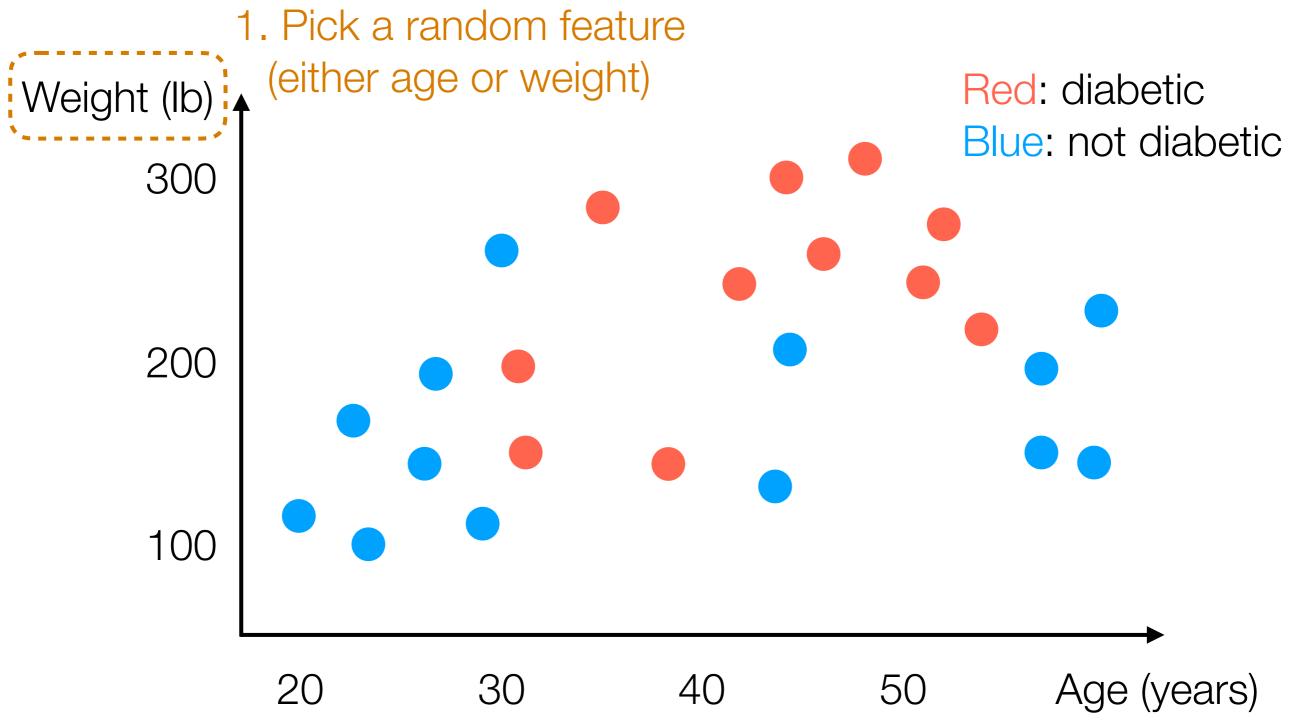
2. Find threshold for which red and blue are as "separate as possible" (on one side, mostly red; on other side, mostly blue)



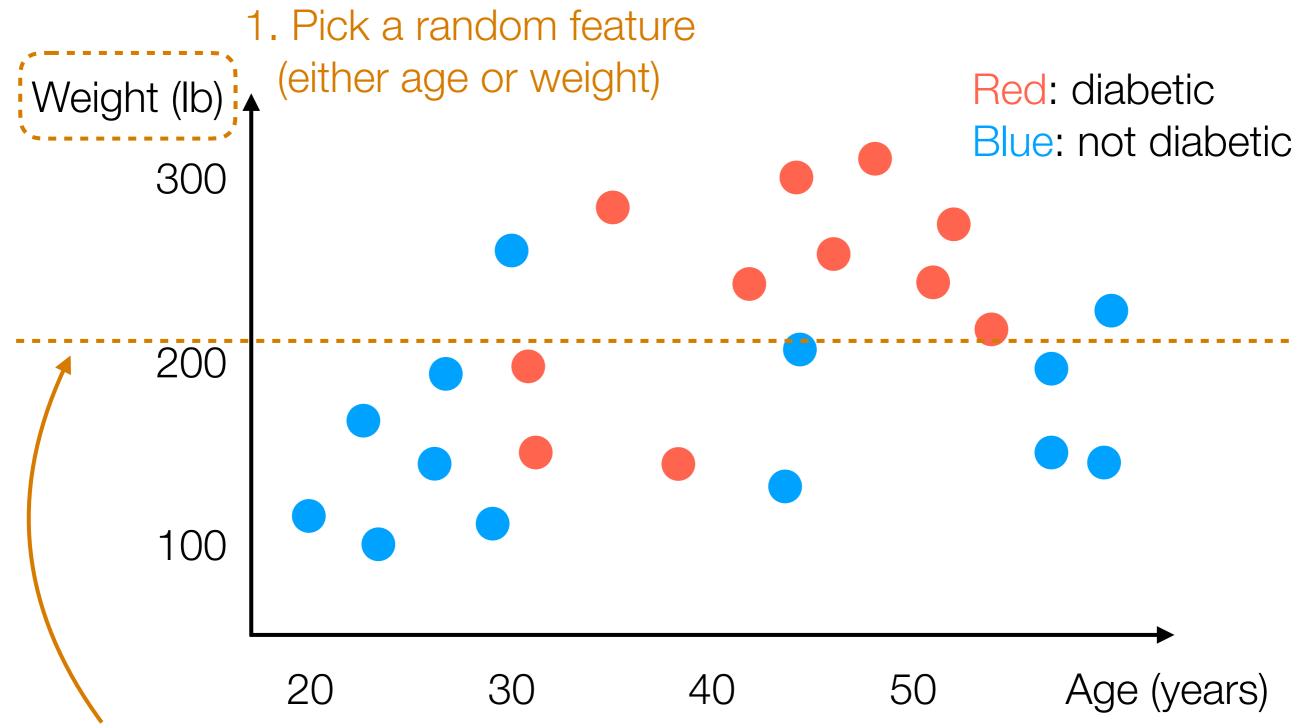
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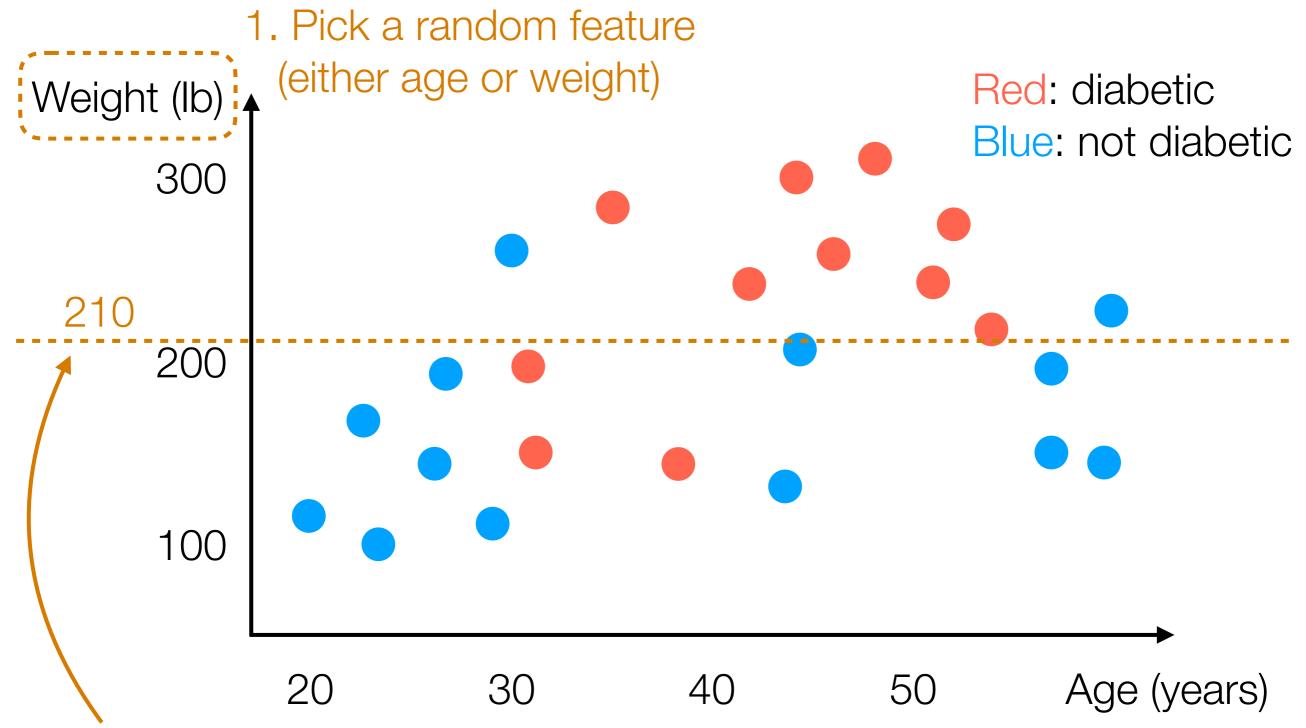
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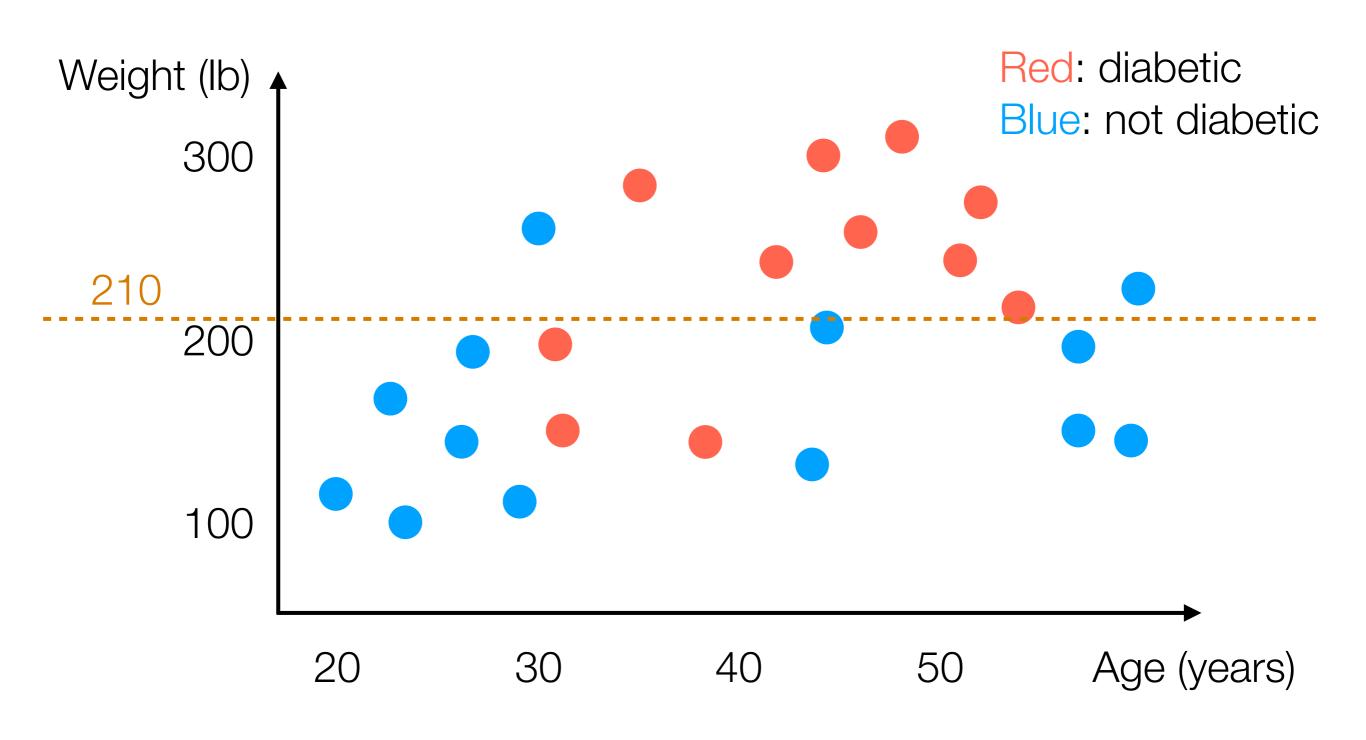
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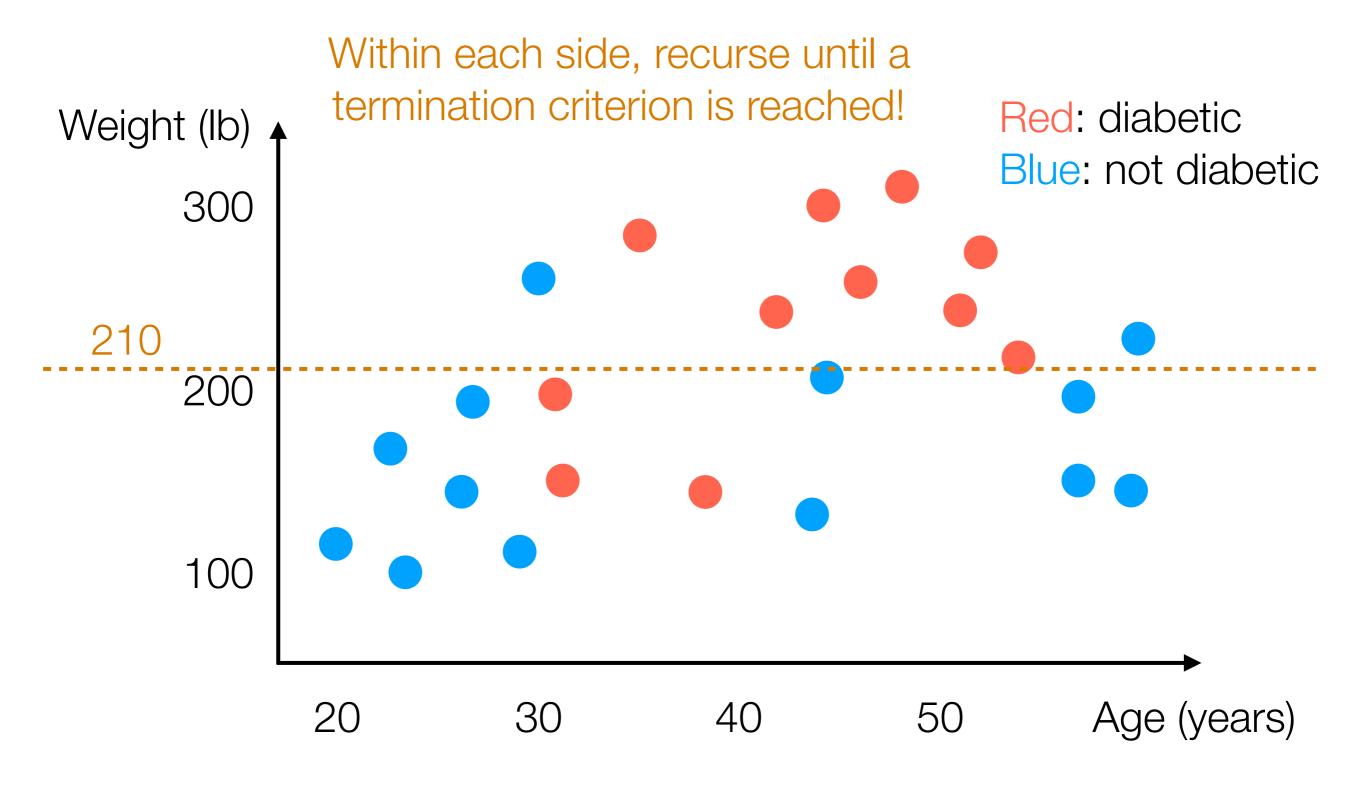


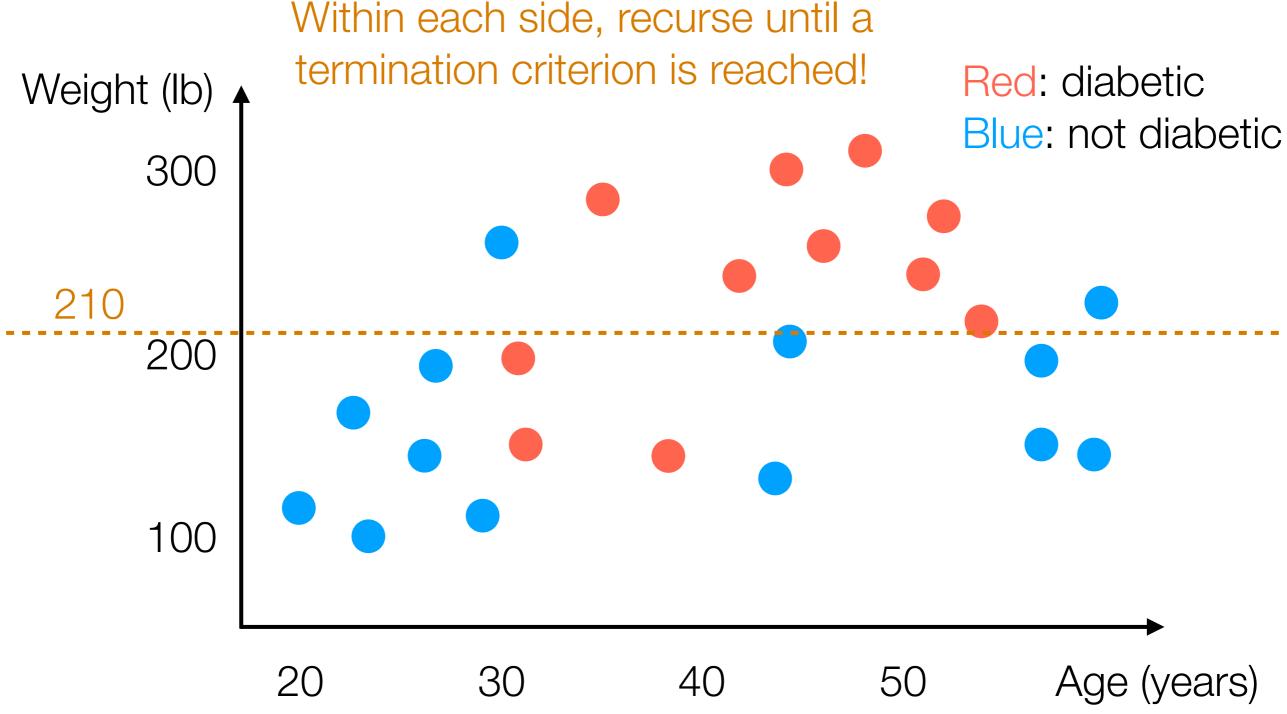
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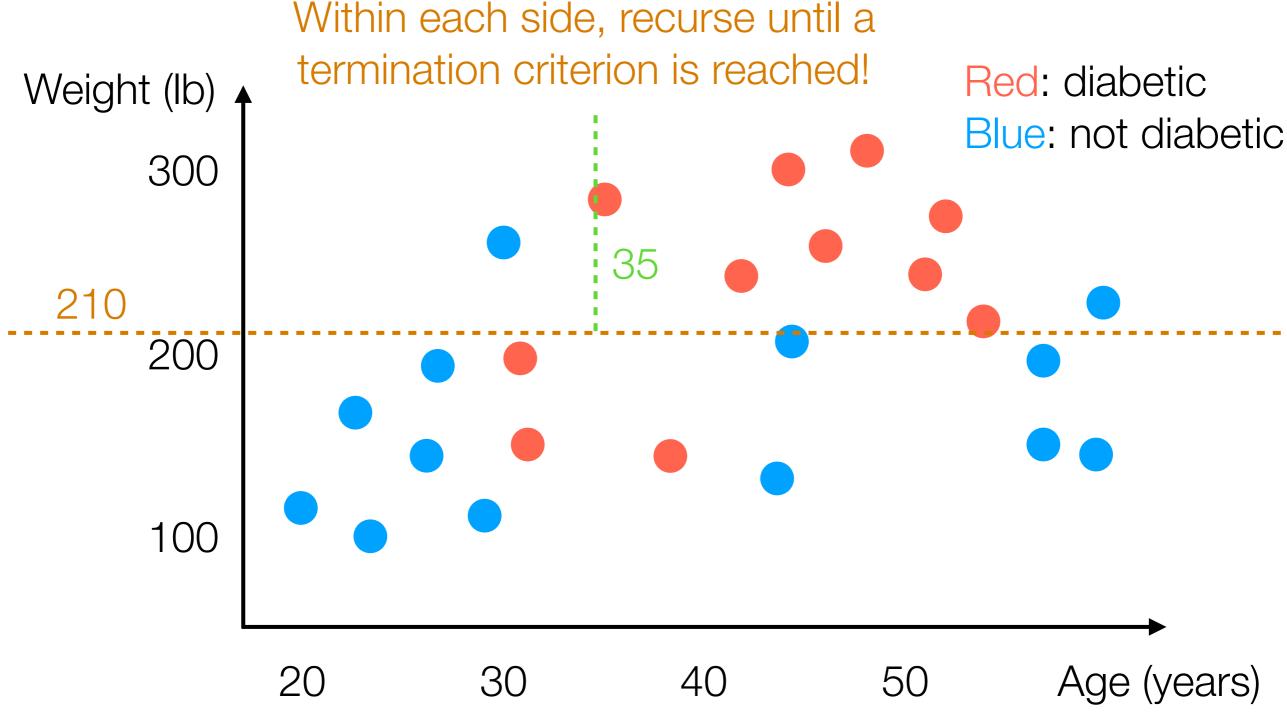


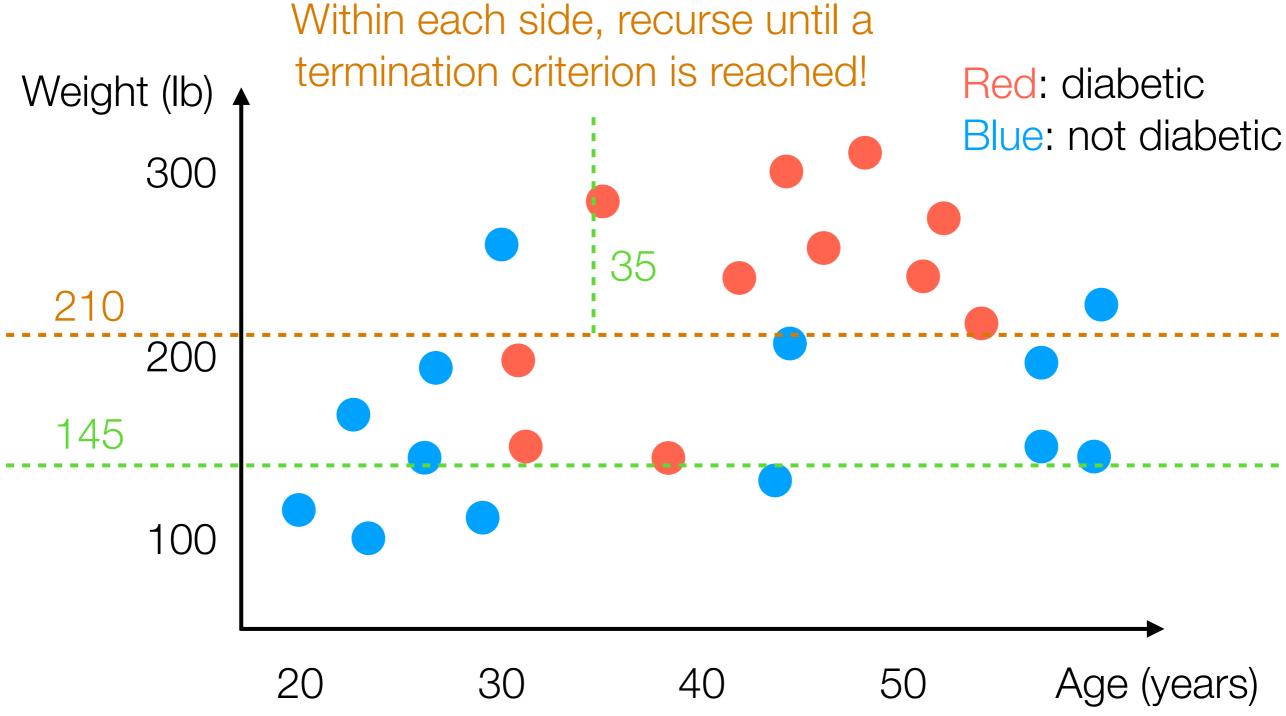
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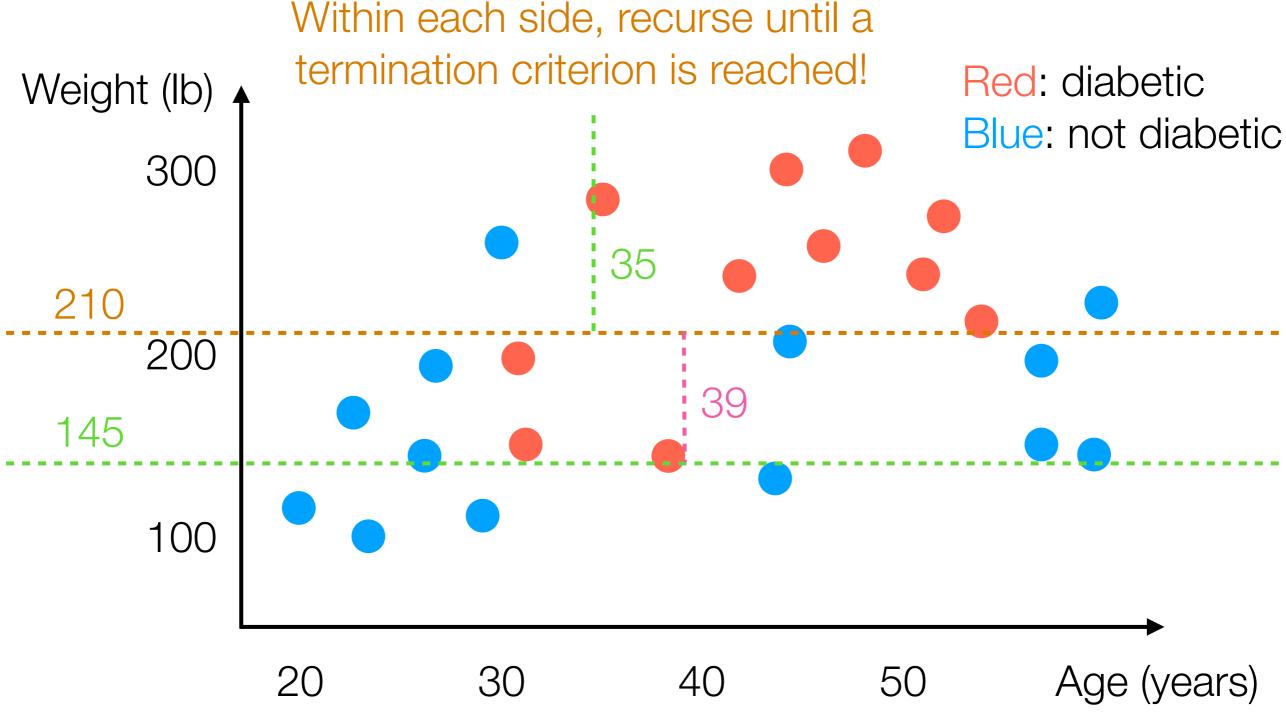


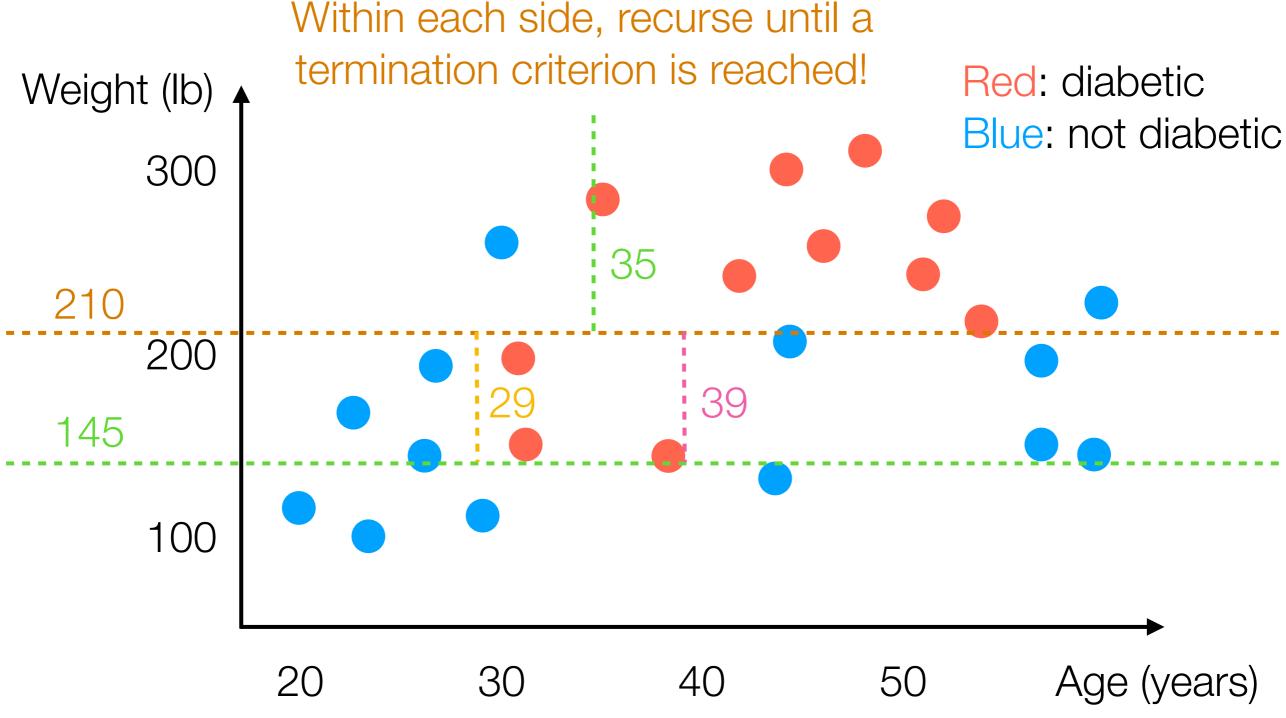


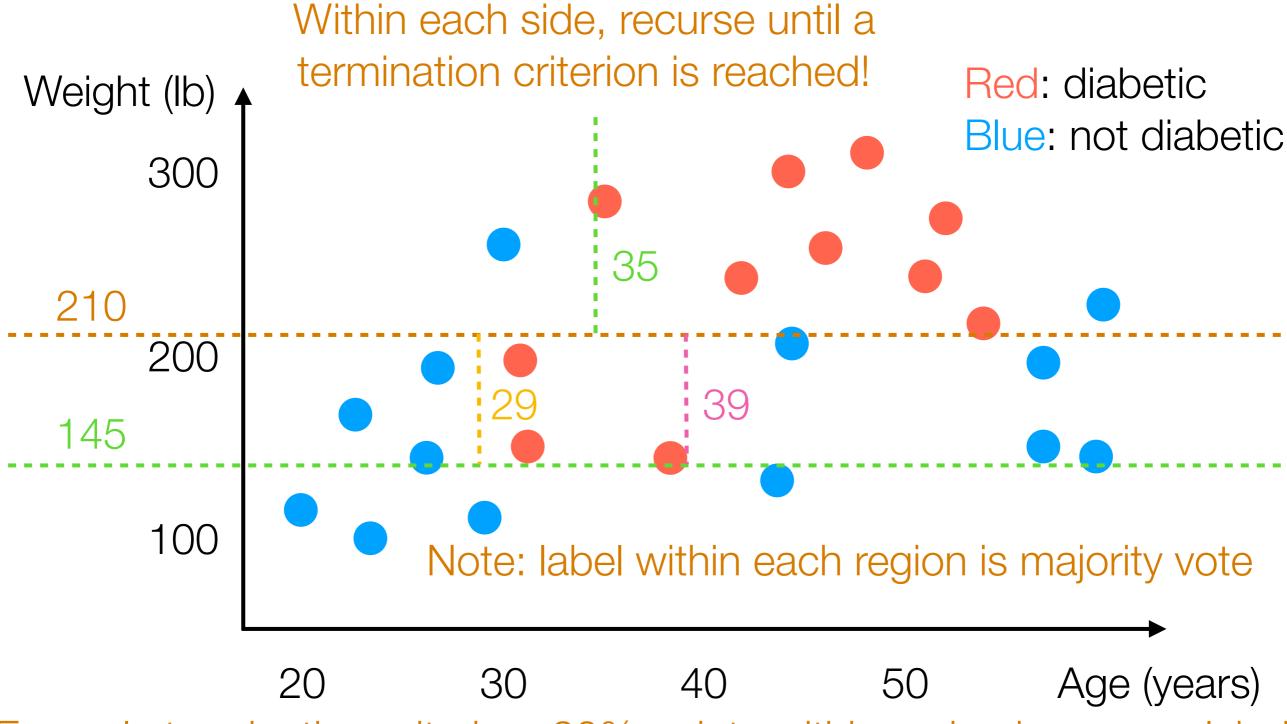


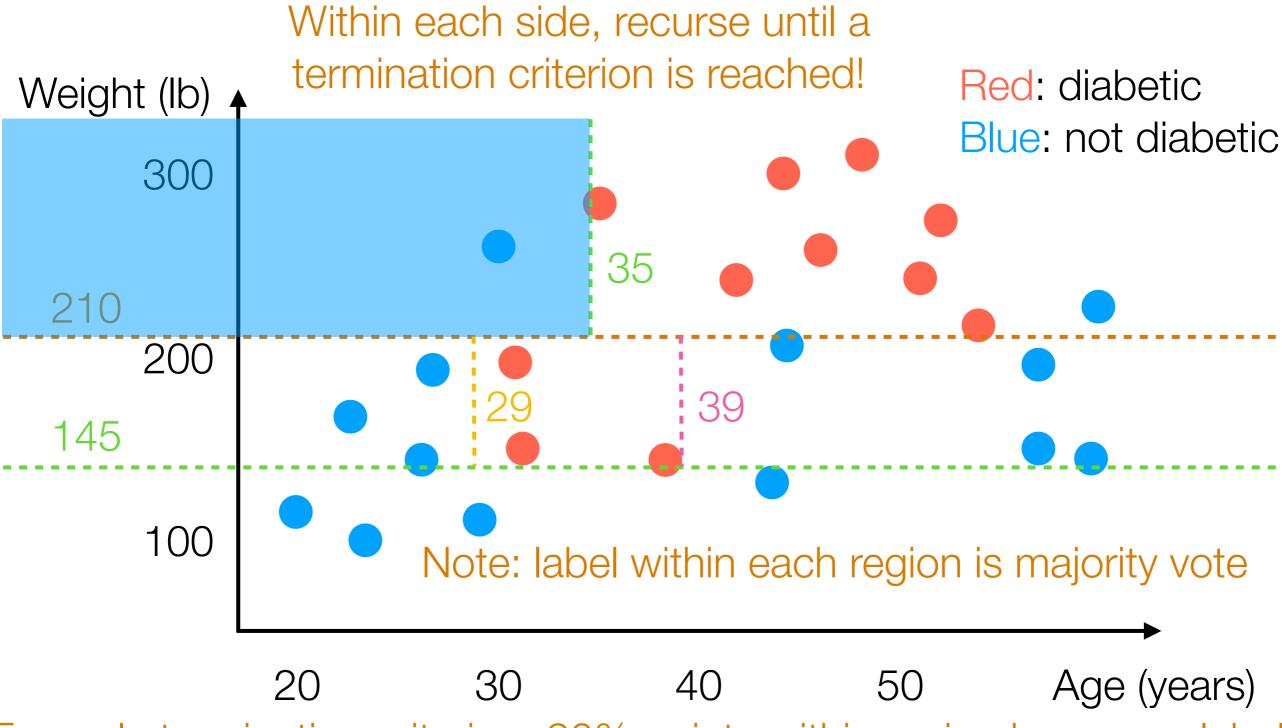


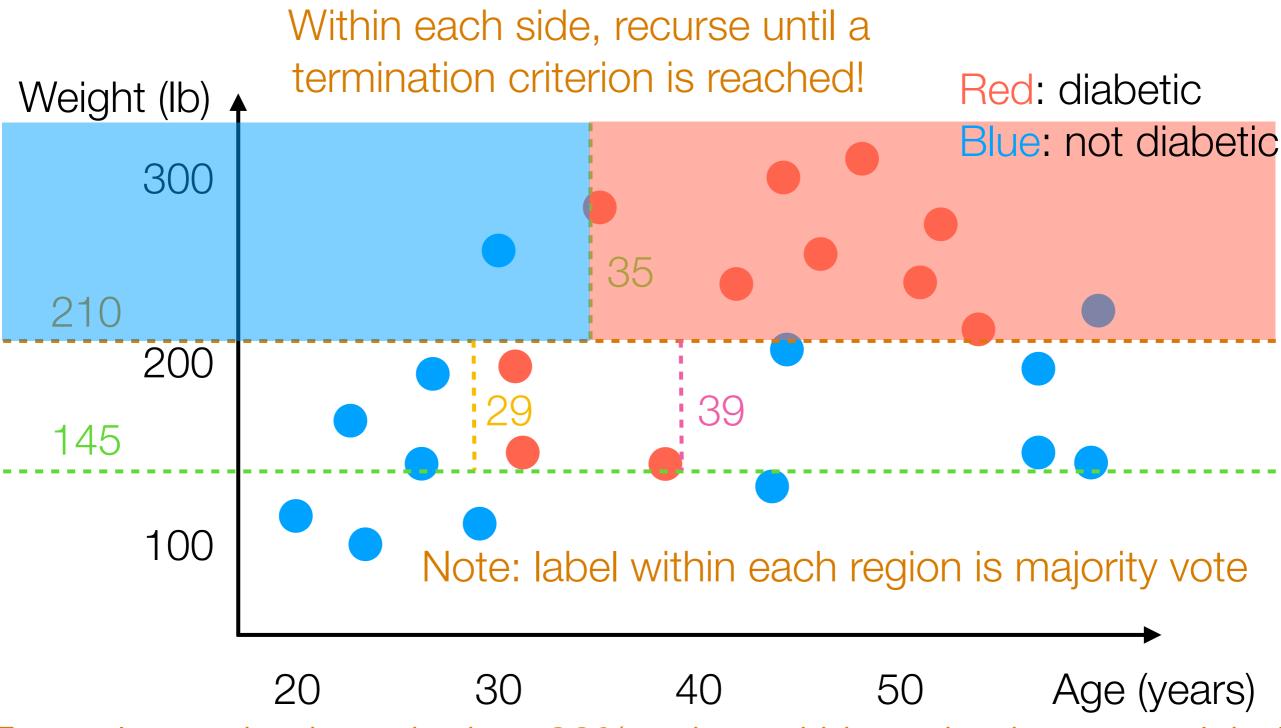


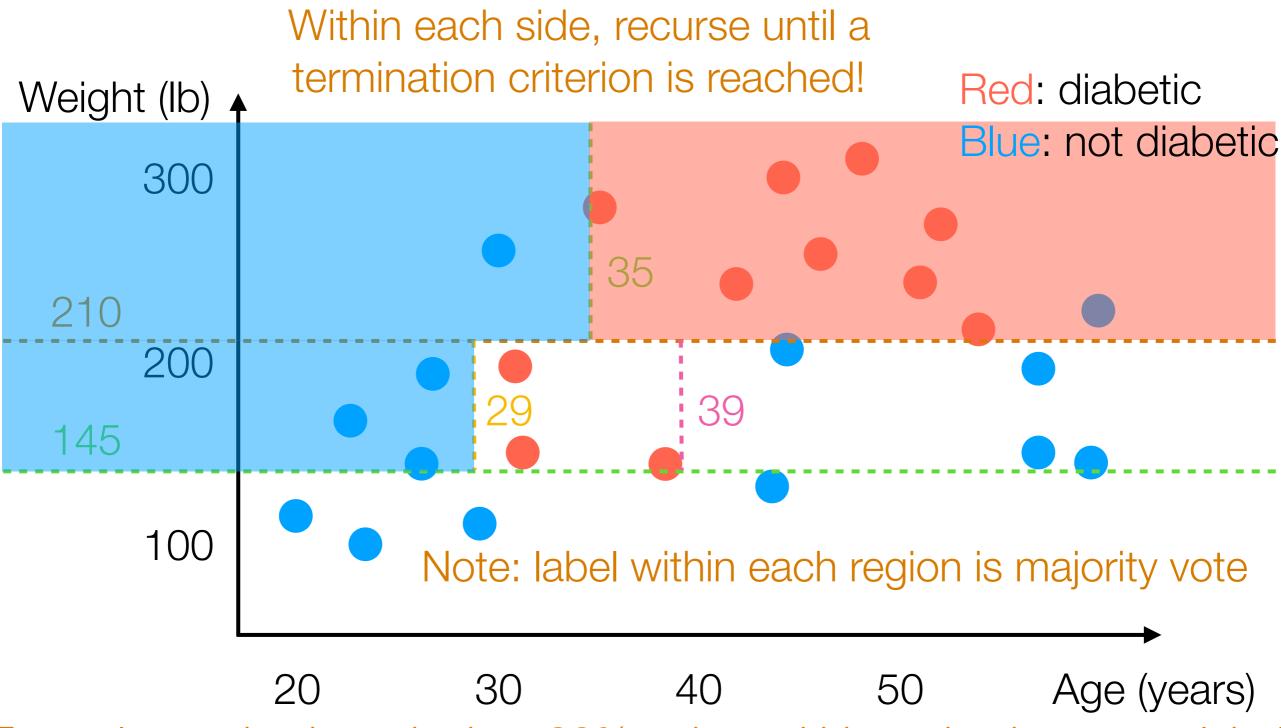


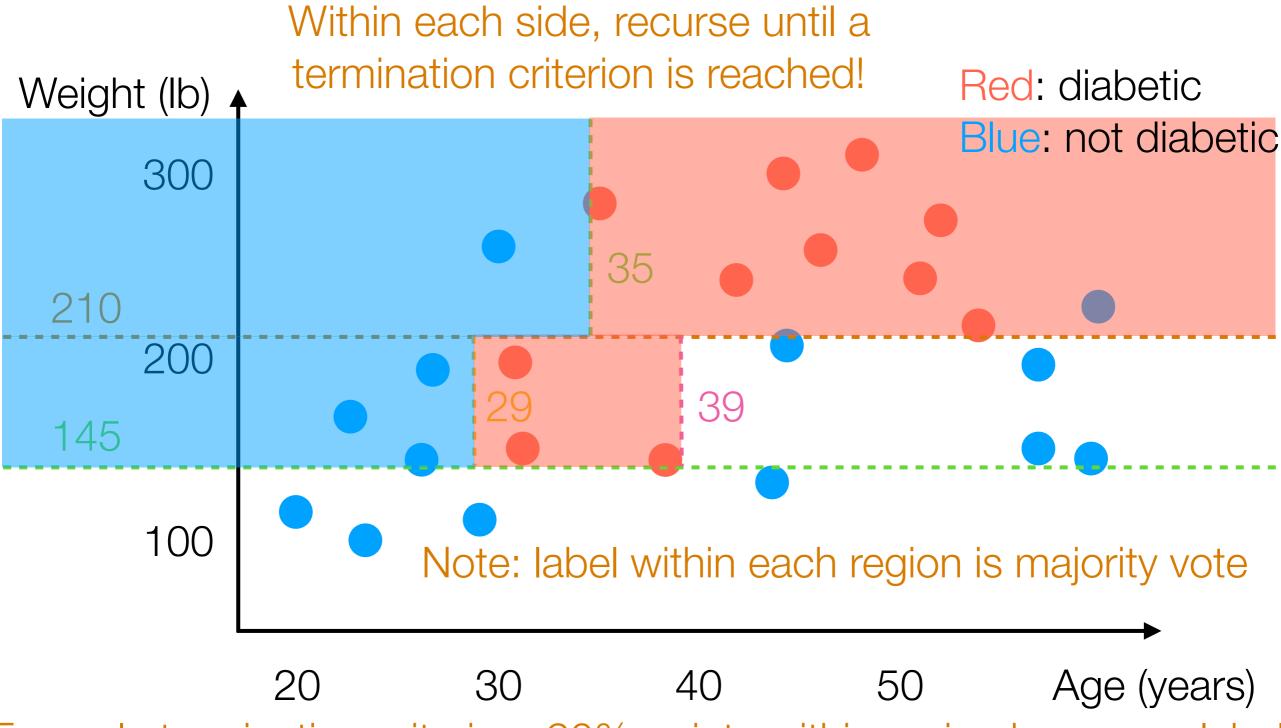


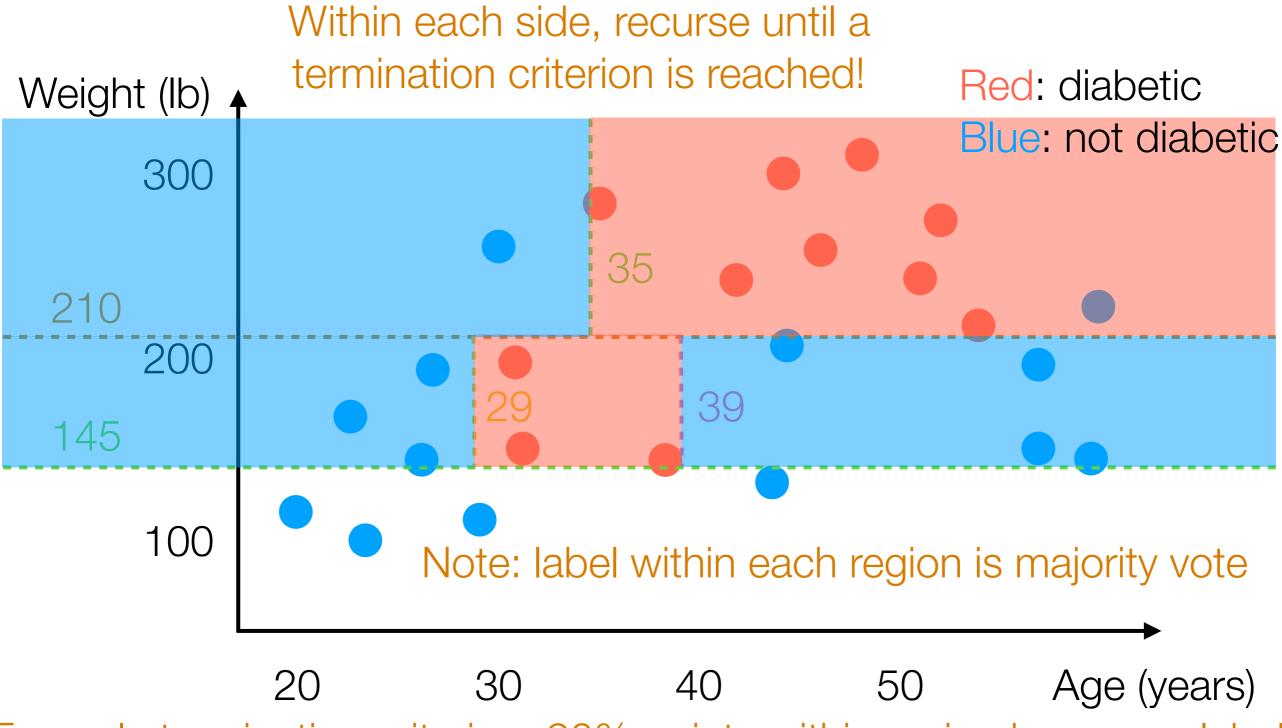


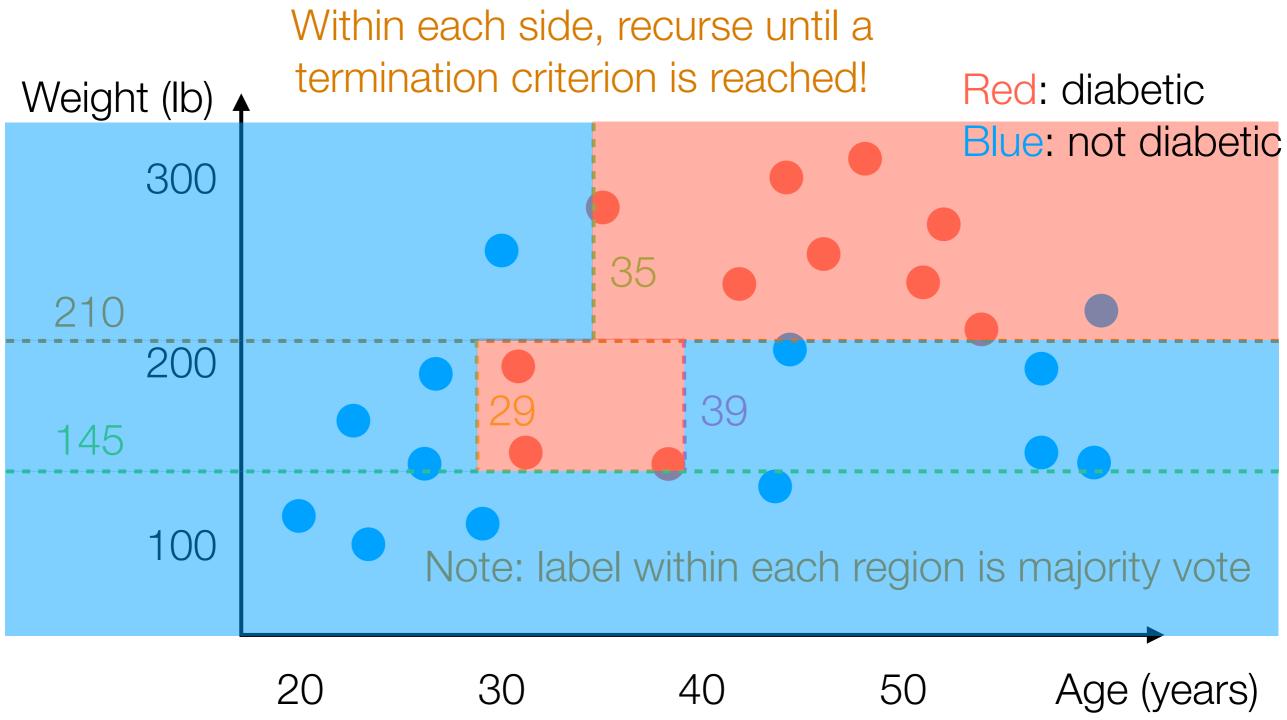


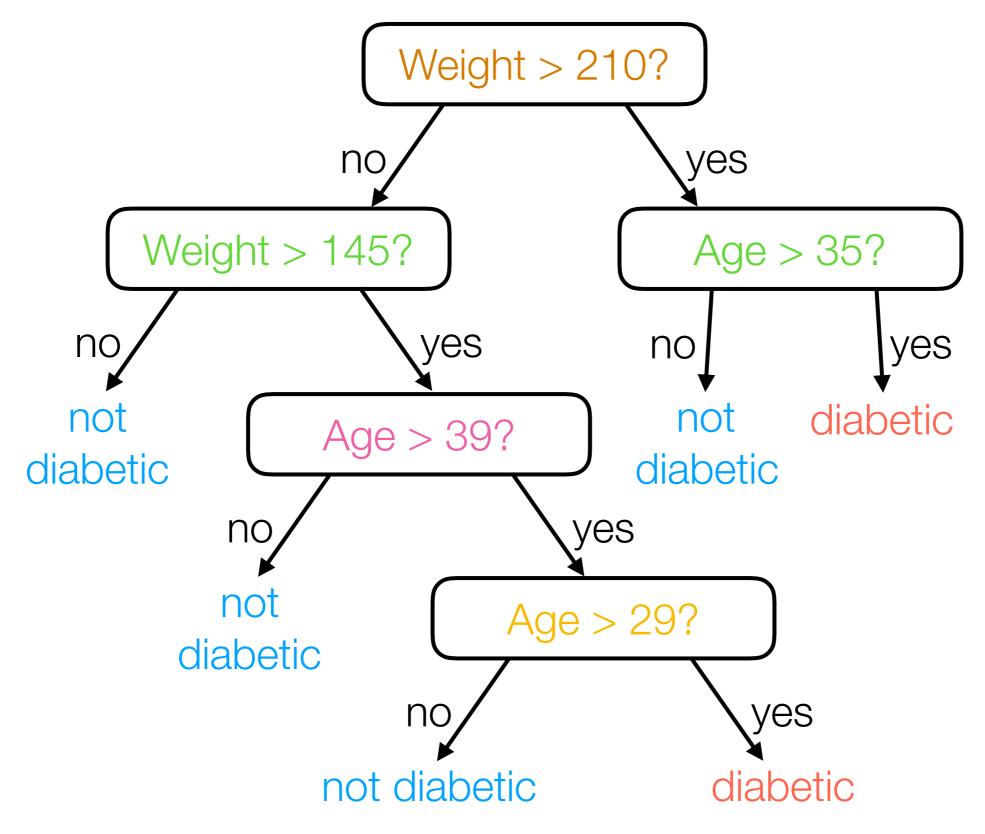


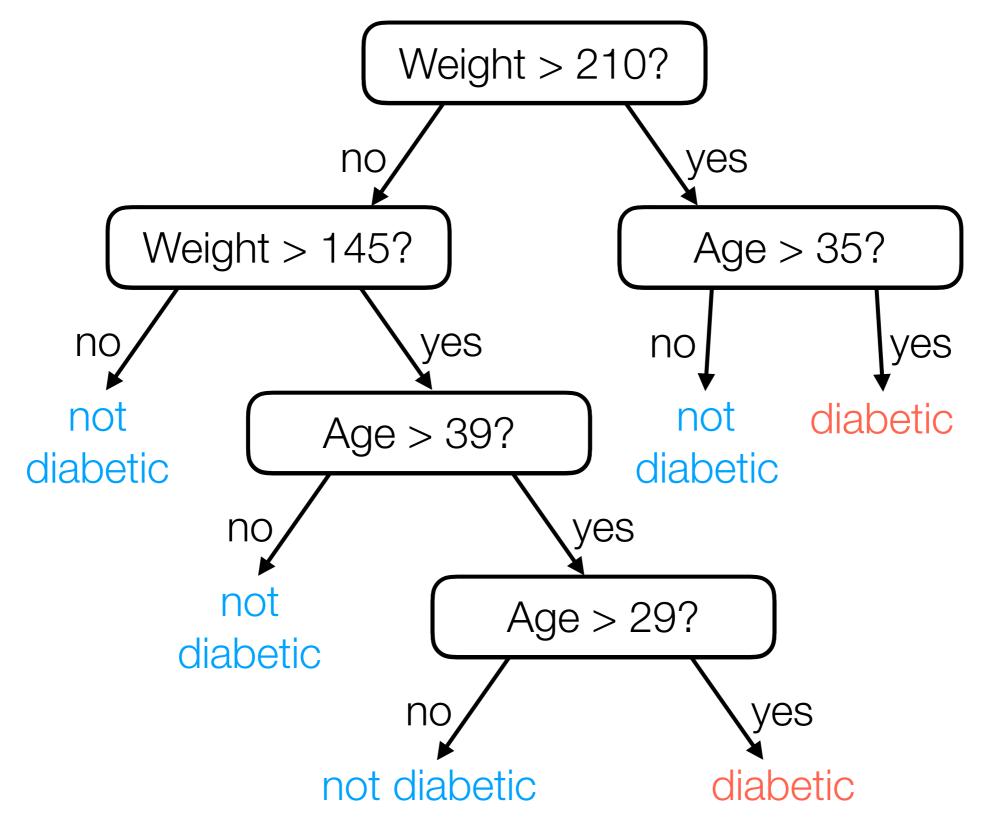


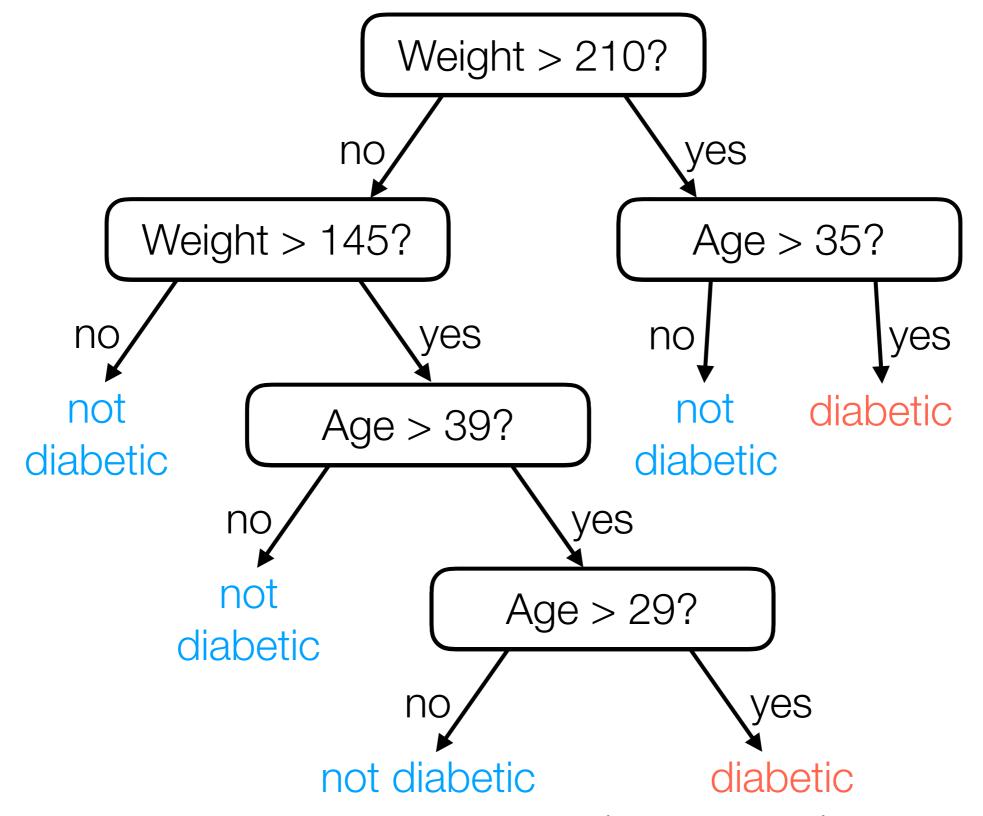






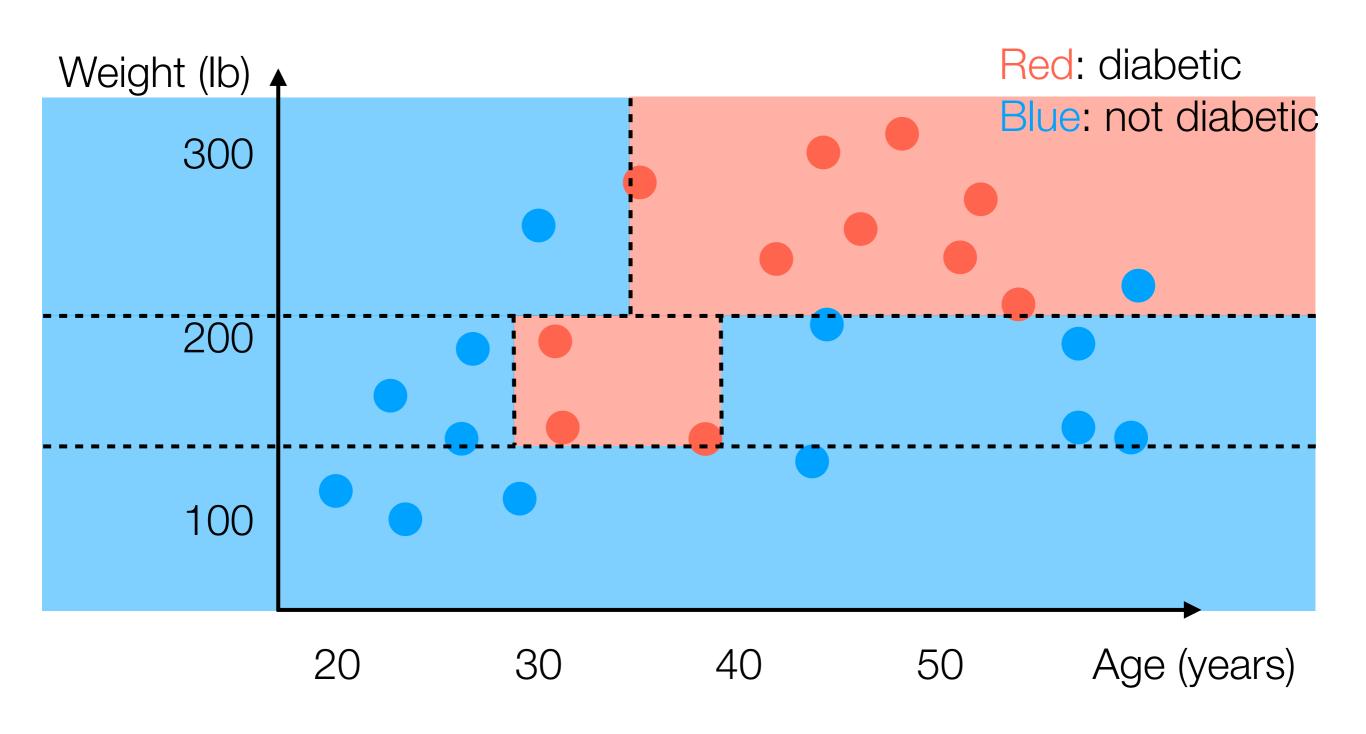






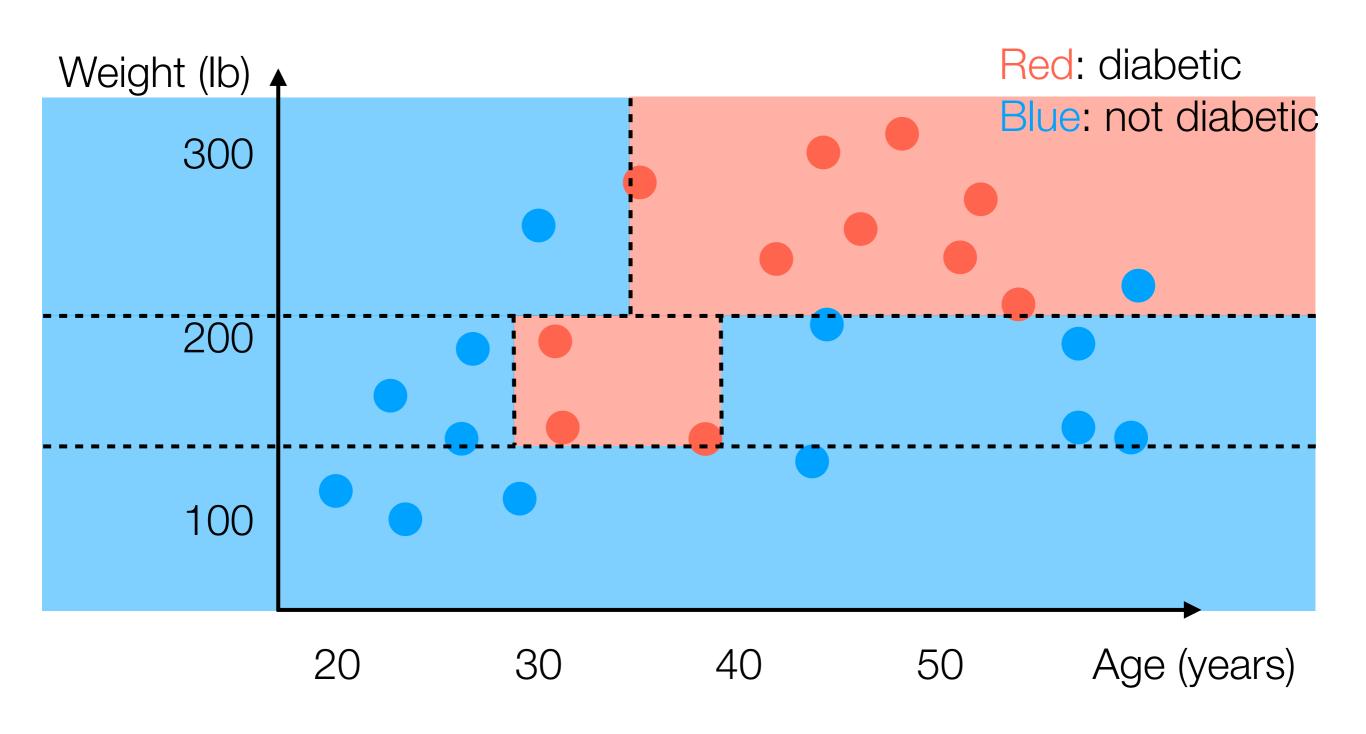
For a new person with feature vector (age, weight), easy to predict!

Nearest Neighbor Interpretation



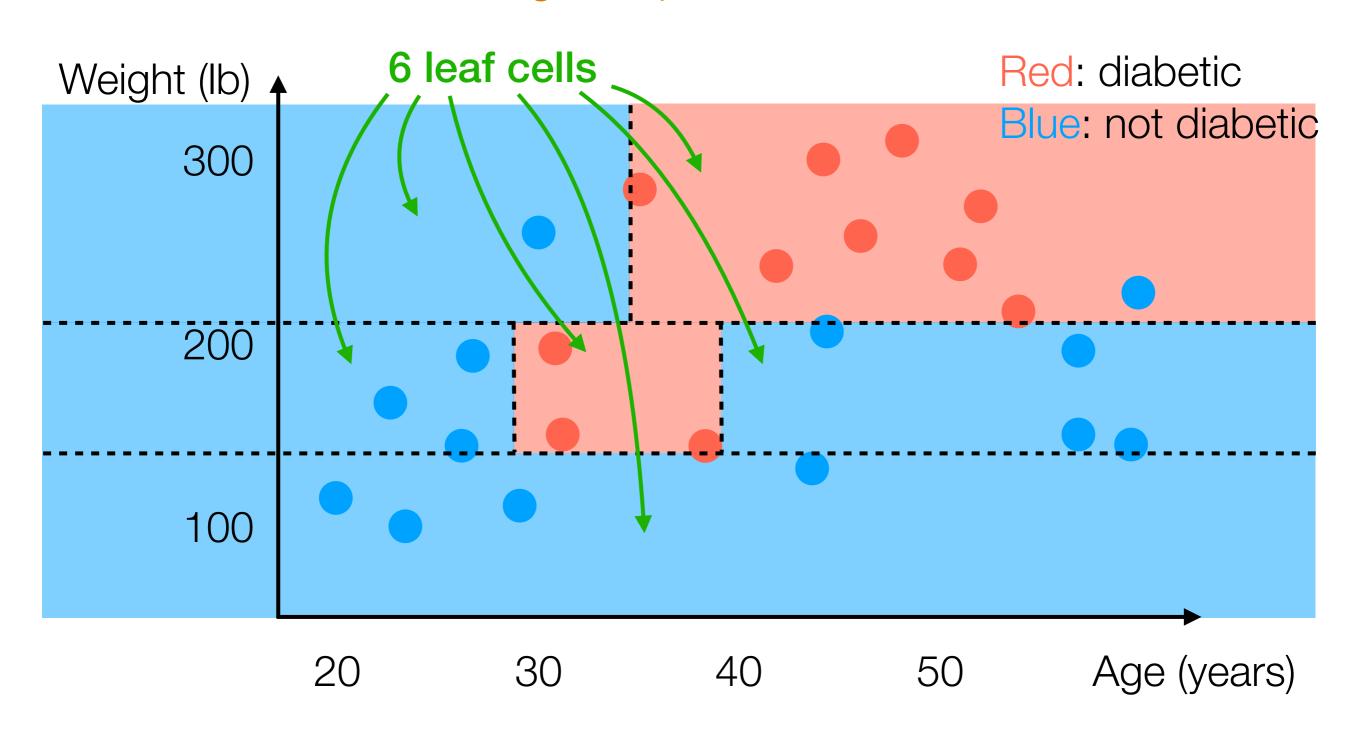
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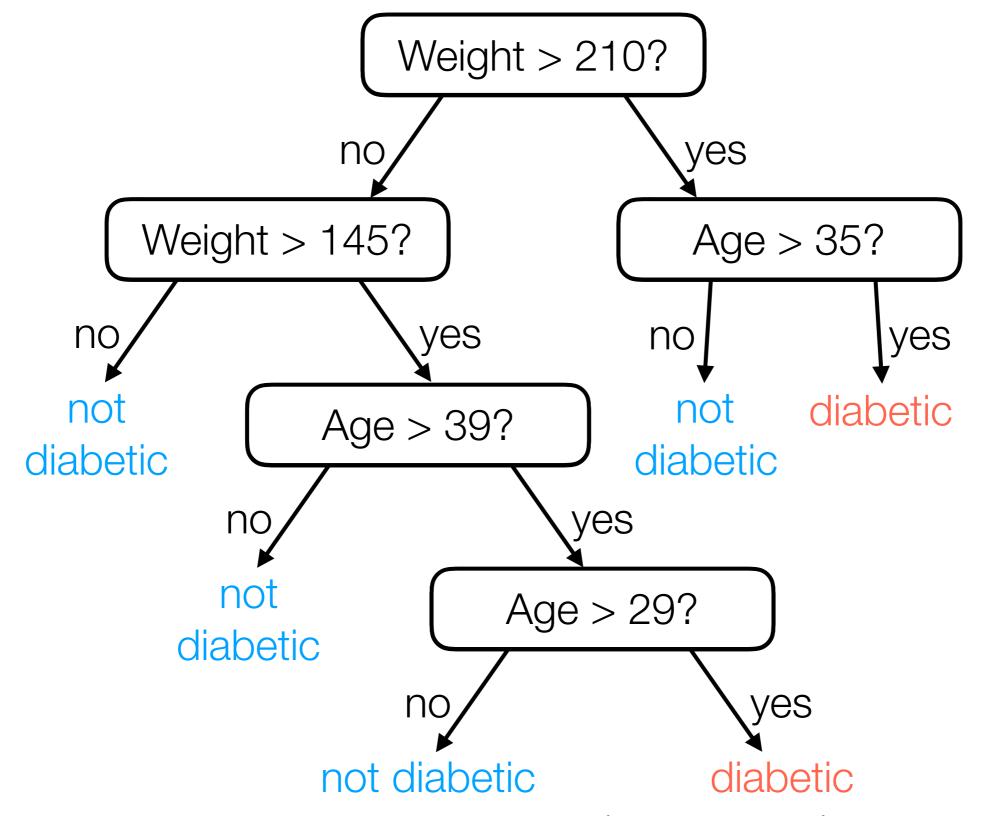
Note: Each training data point lands in one "leaf cell"



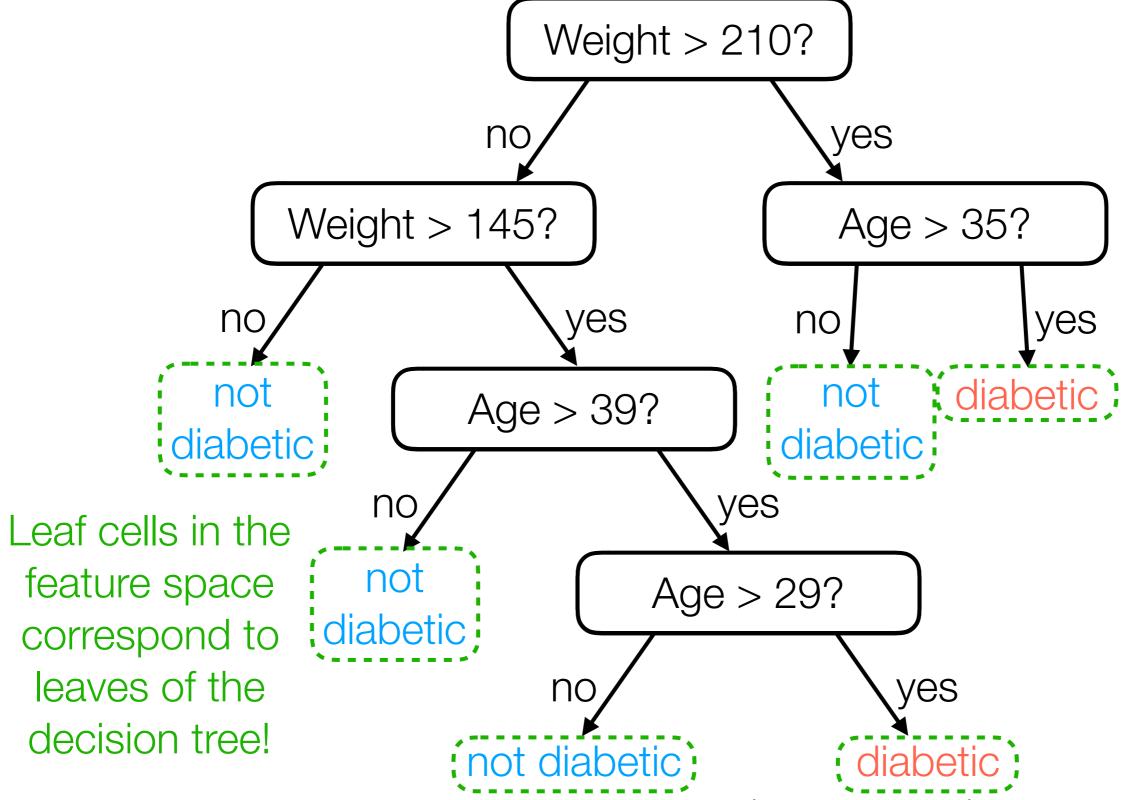
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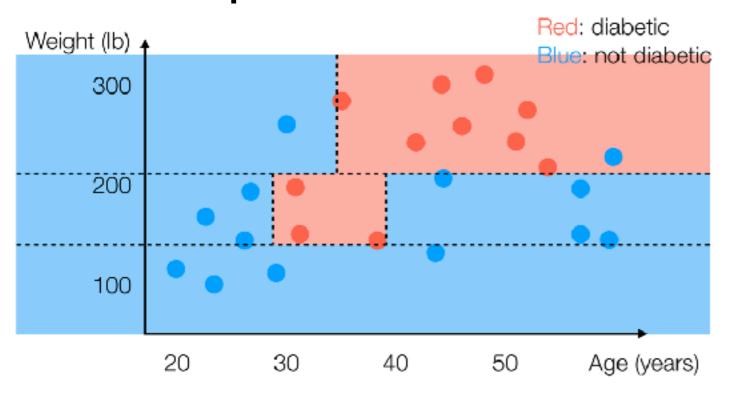


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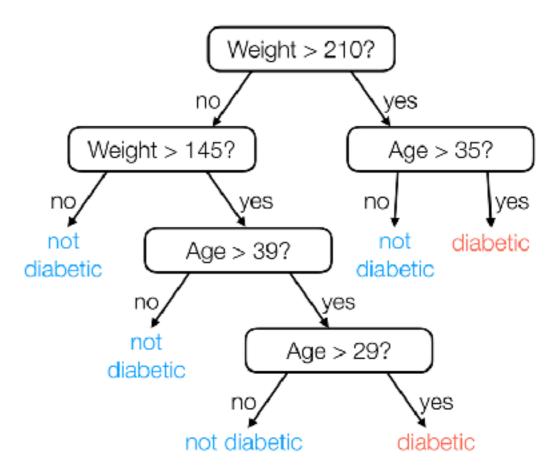


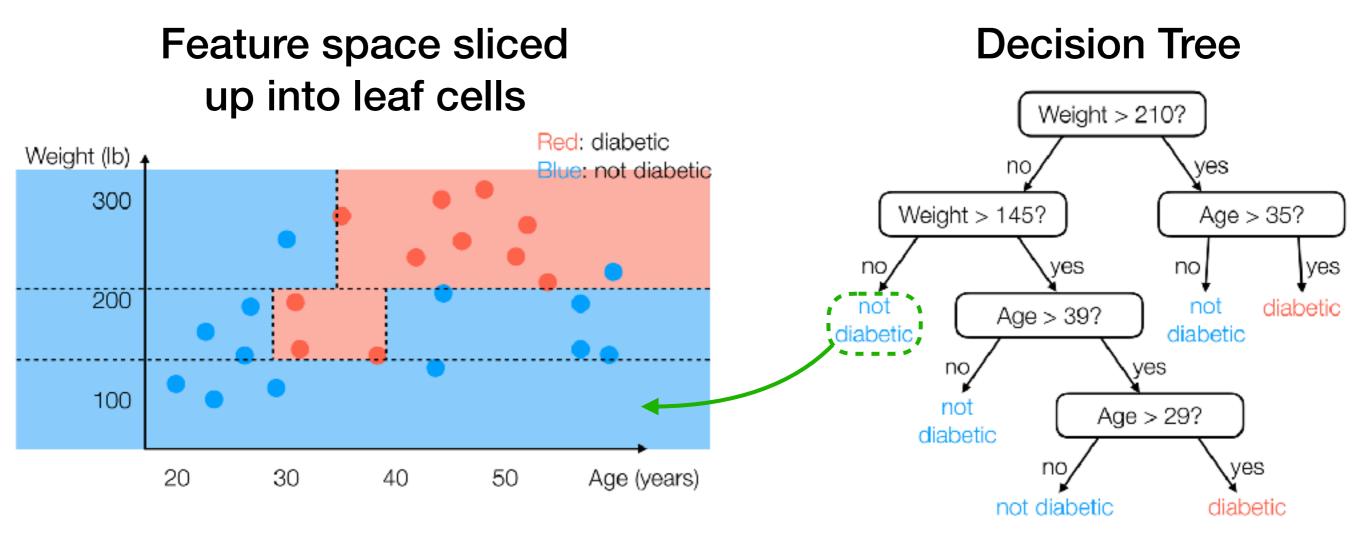
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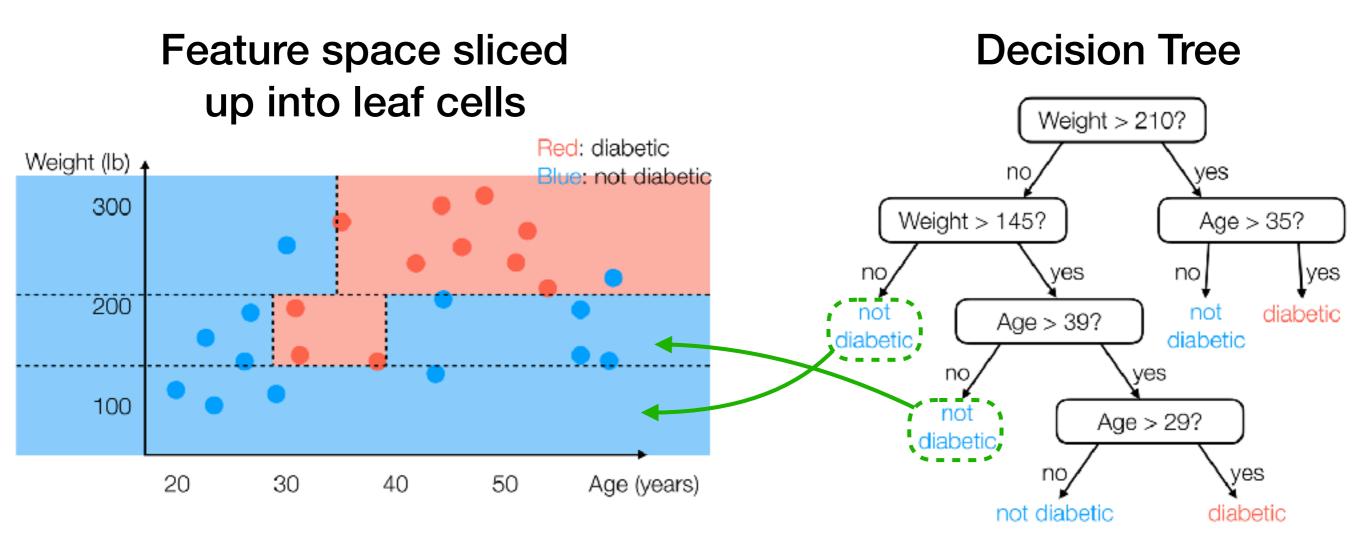
Feature space sliced up into leaf cells

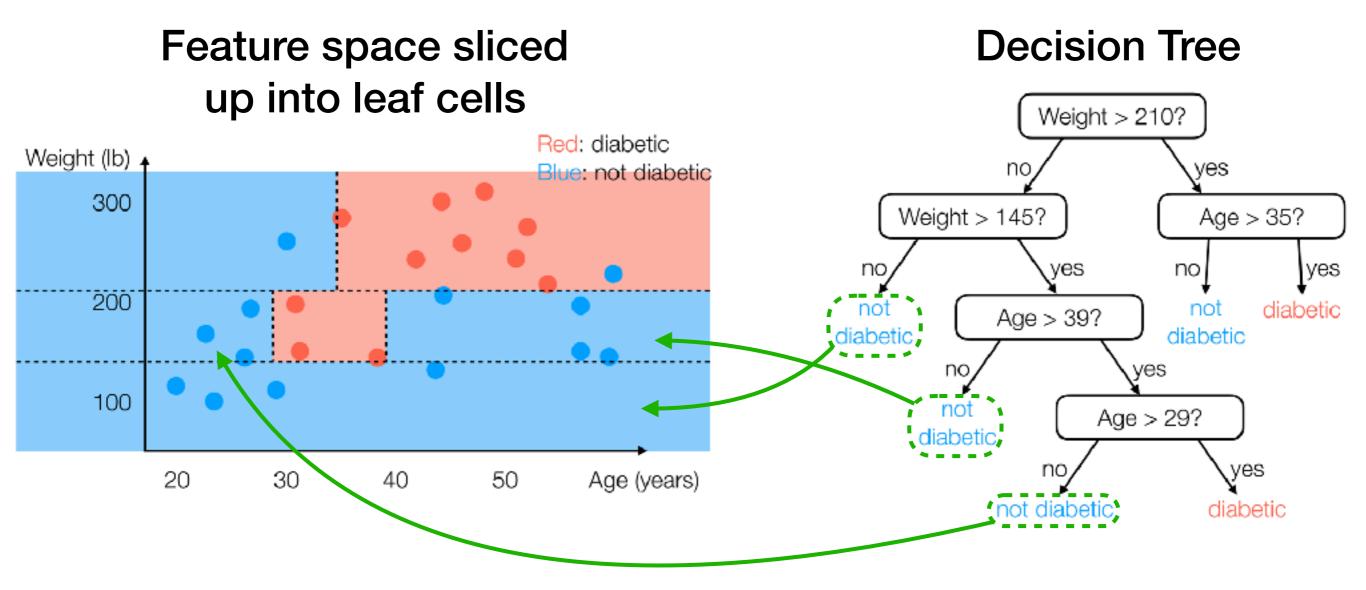


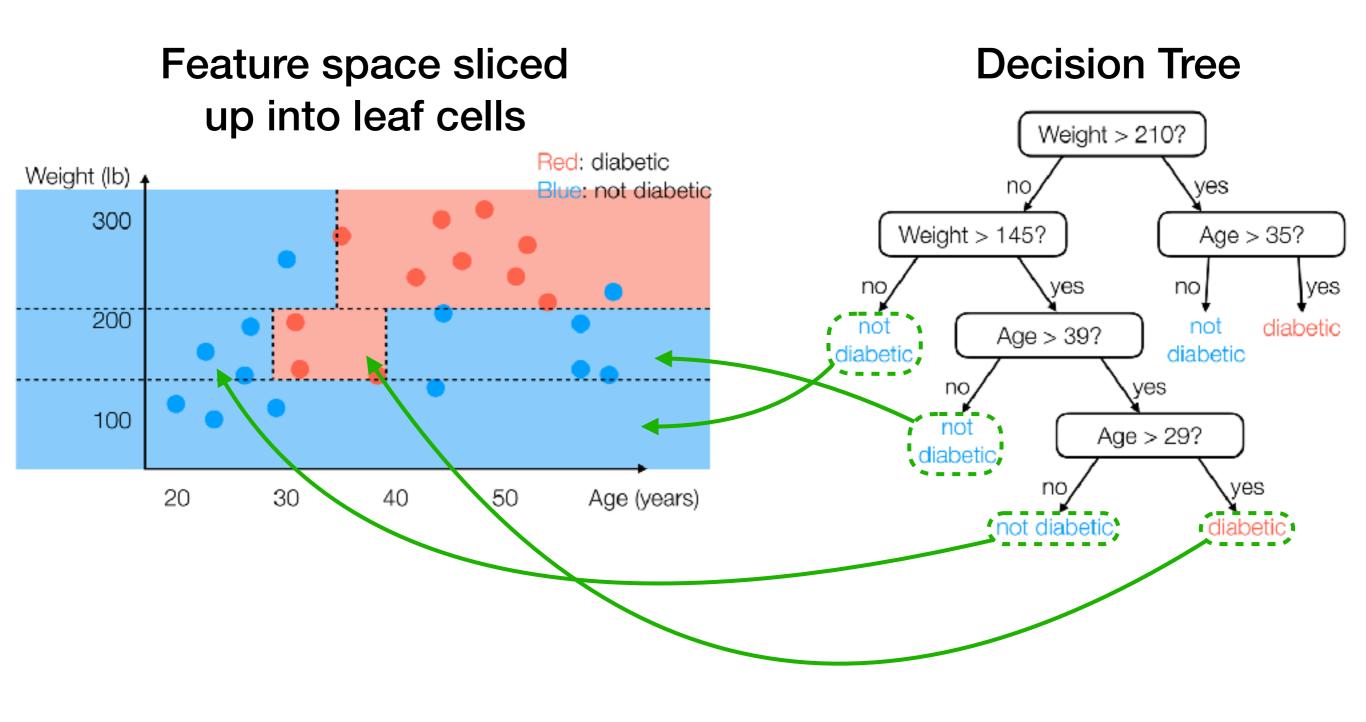
Decision Tree

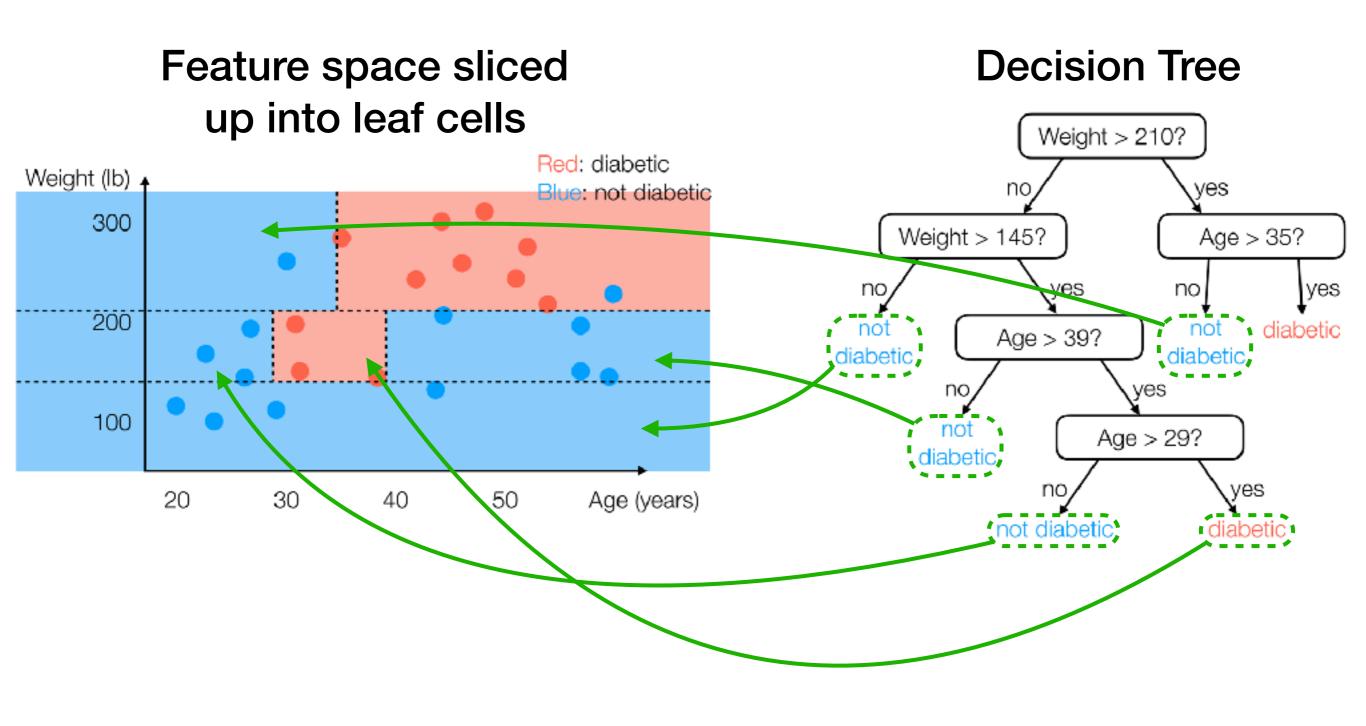


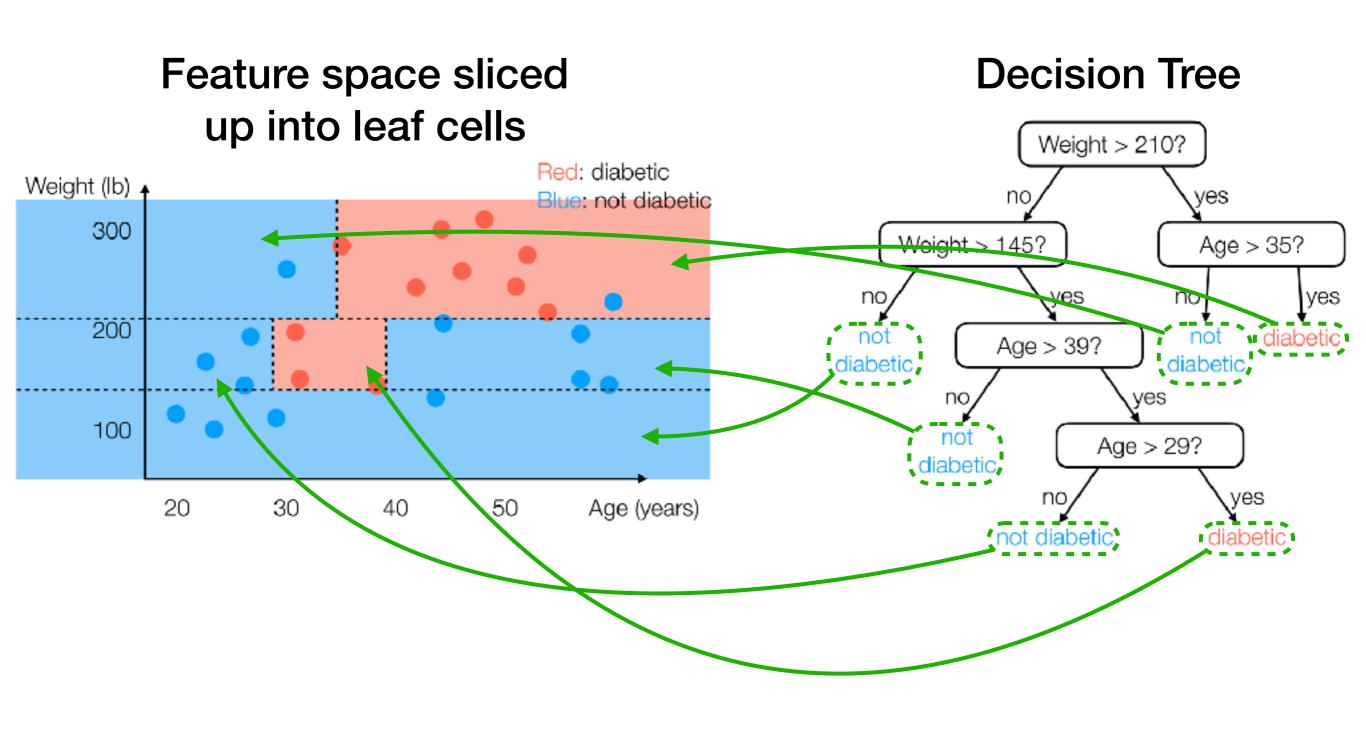




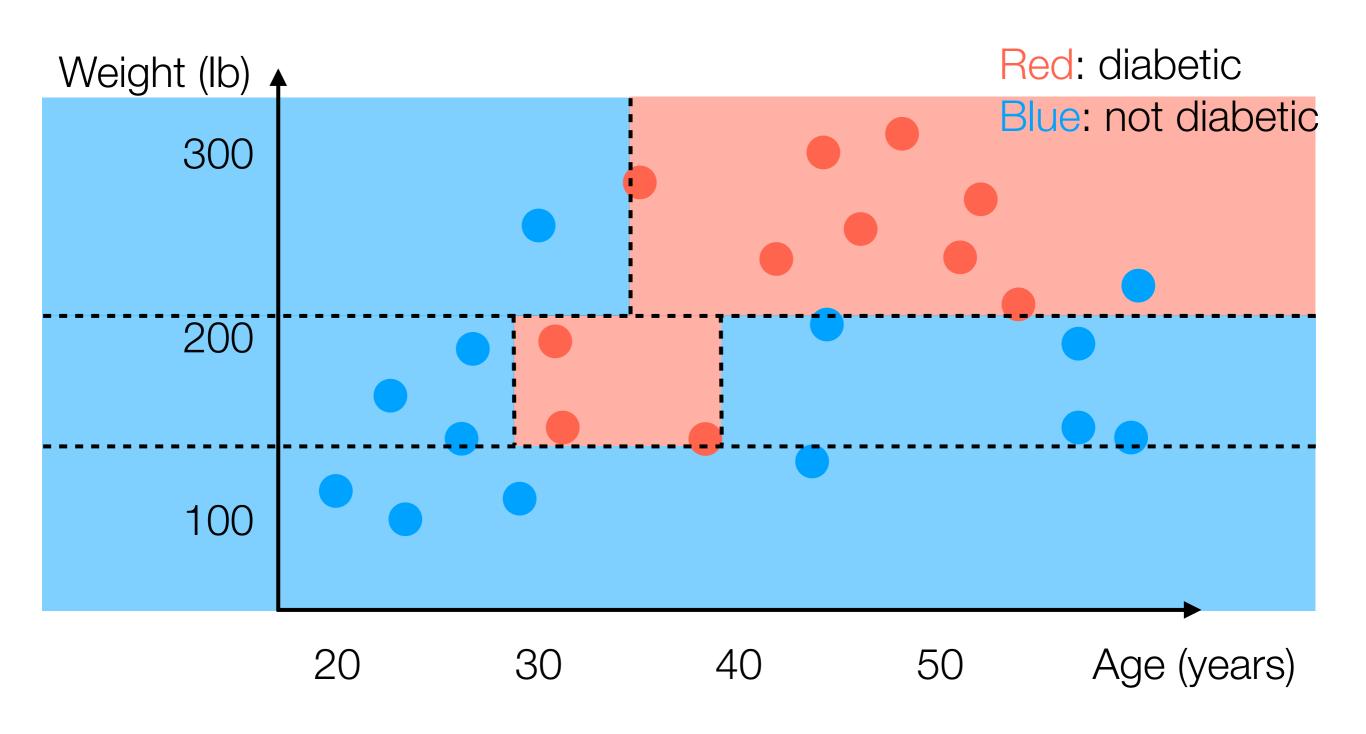




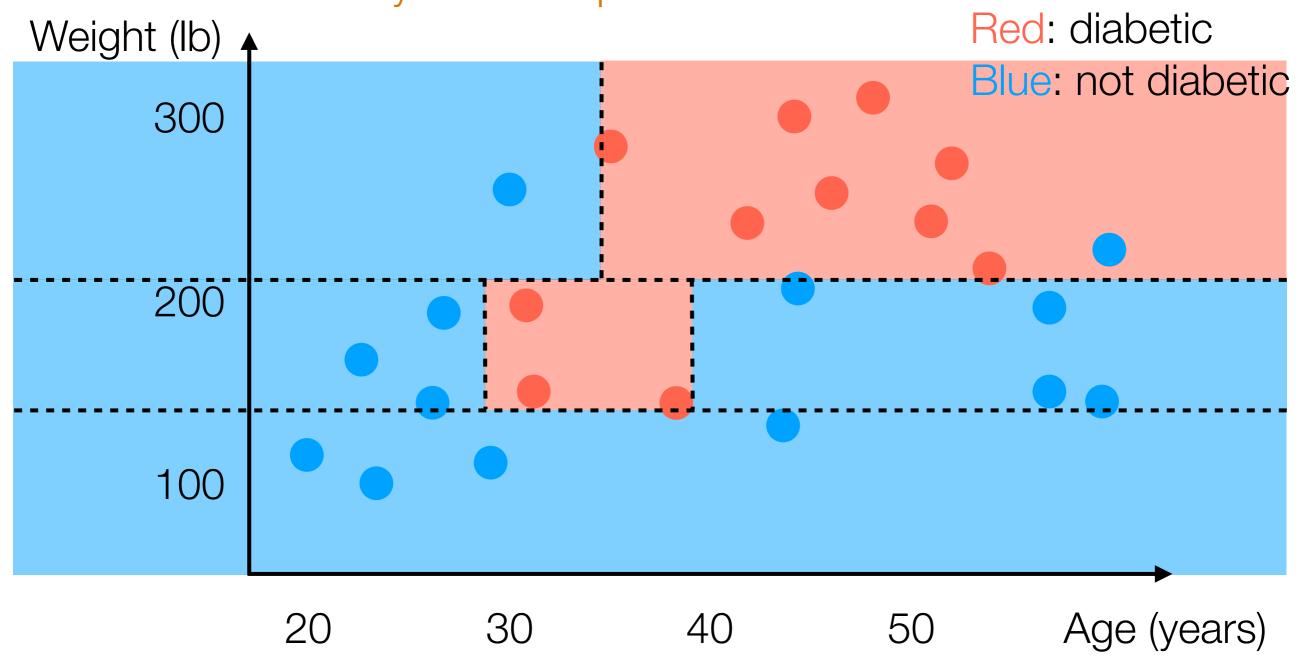




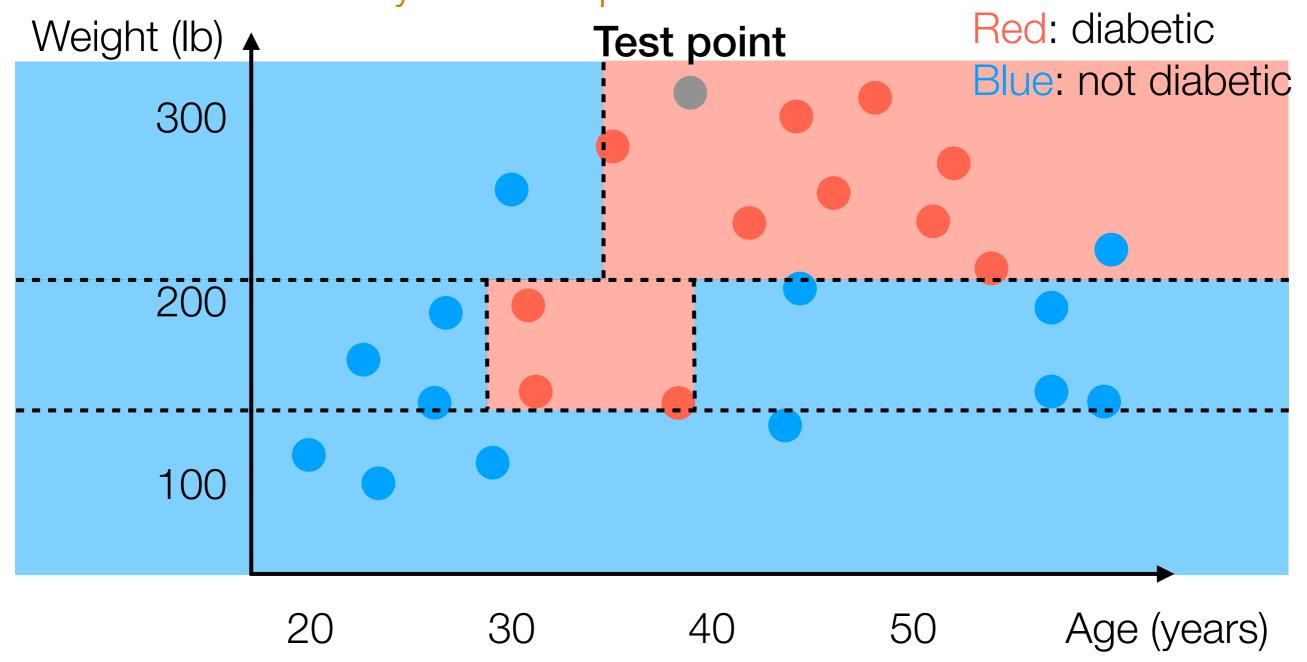
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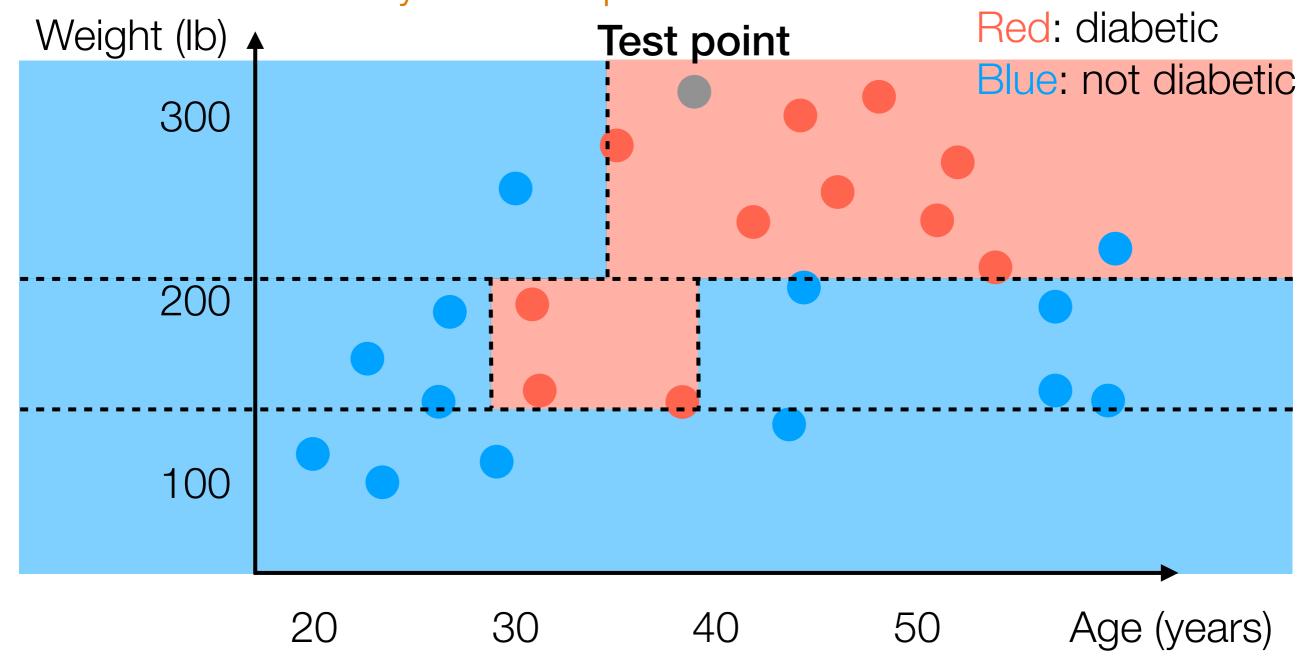
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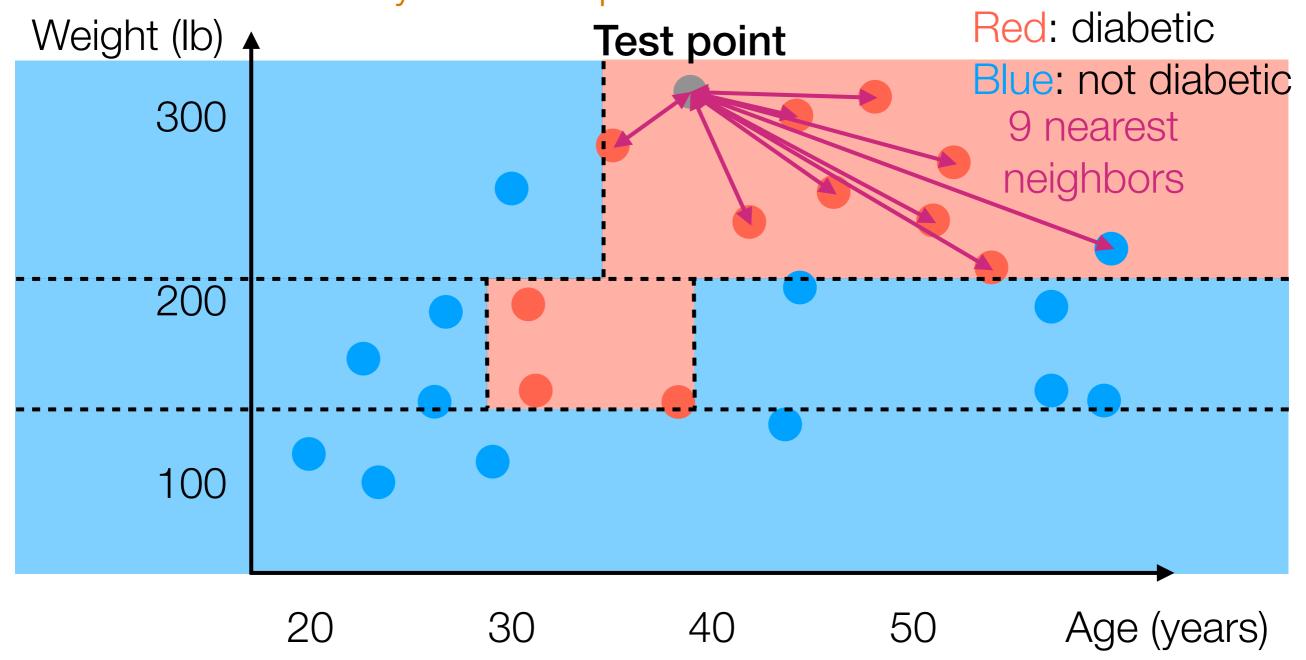
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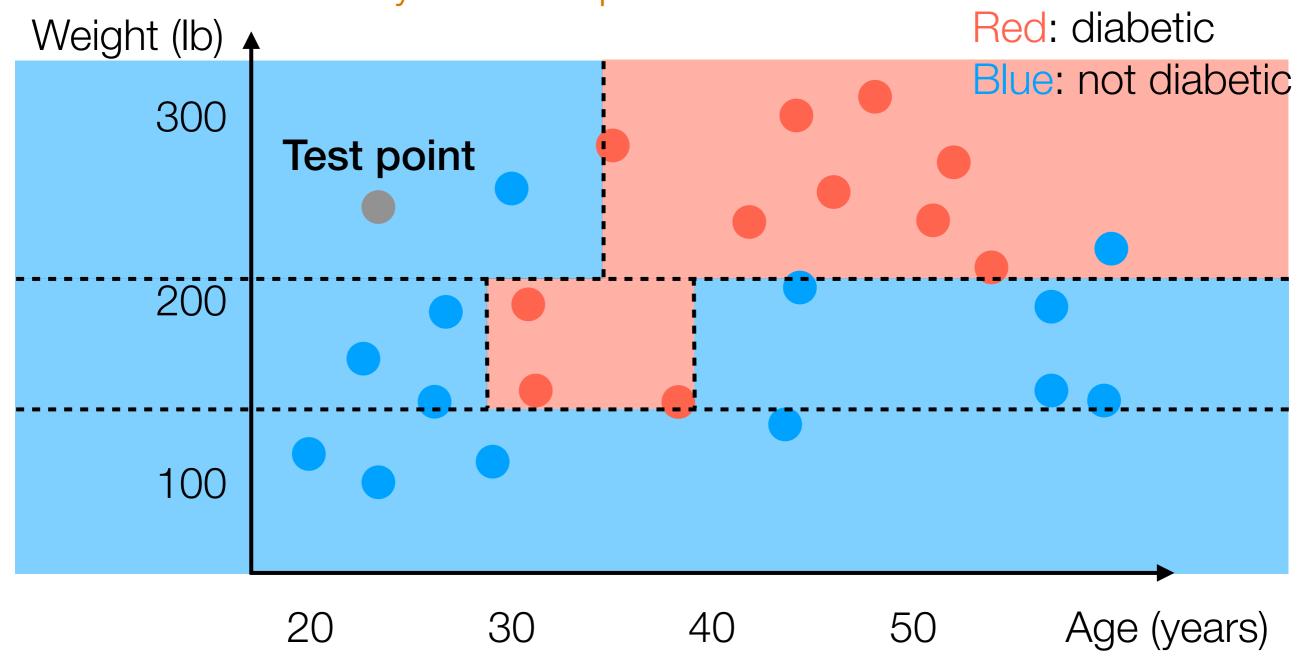
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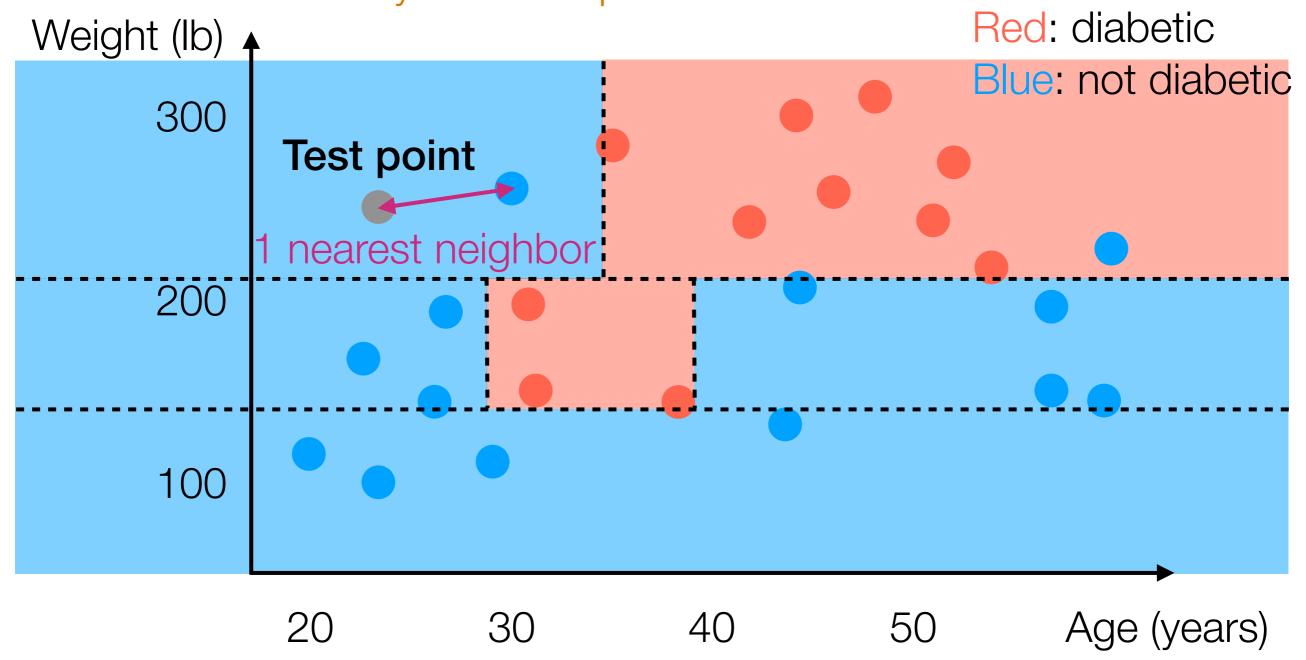
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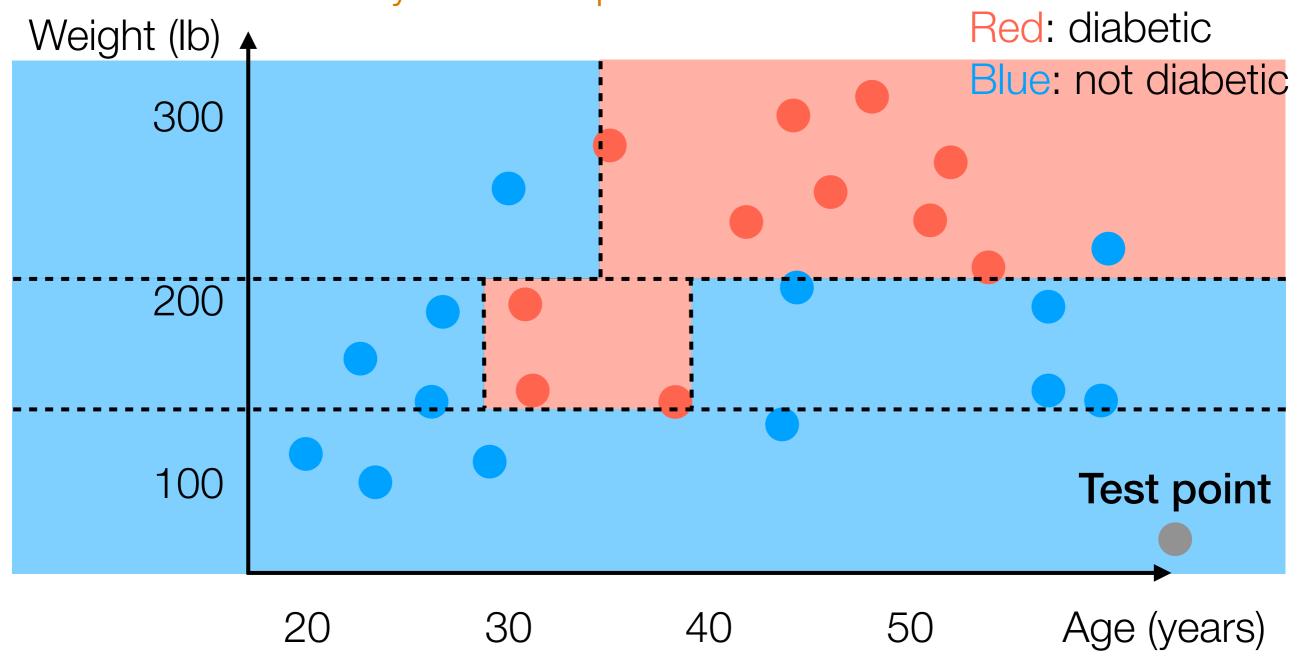
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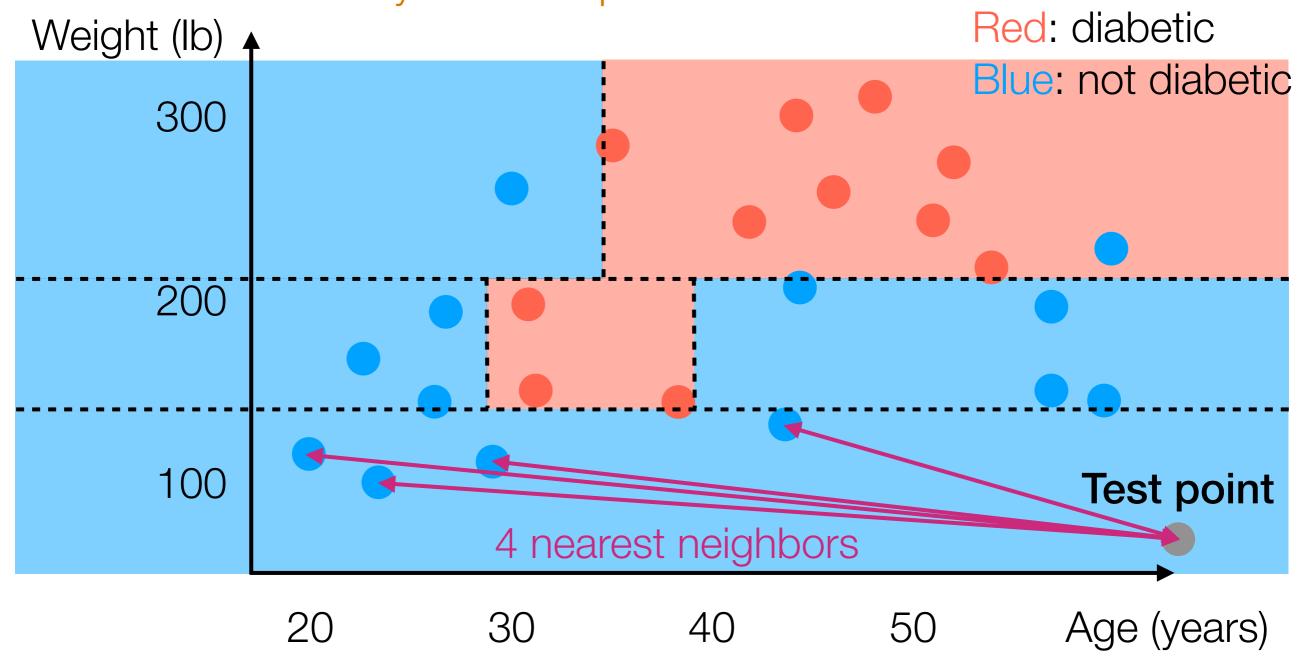
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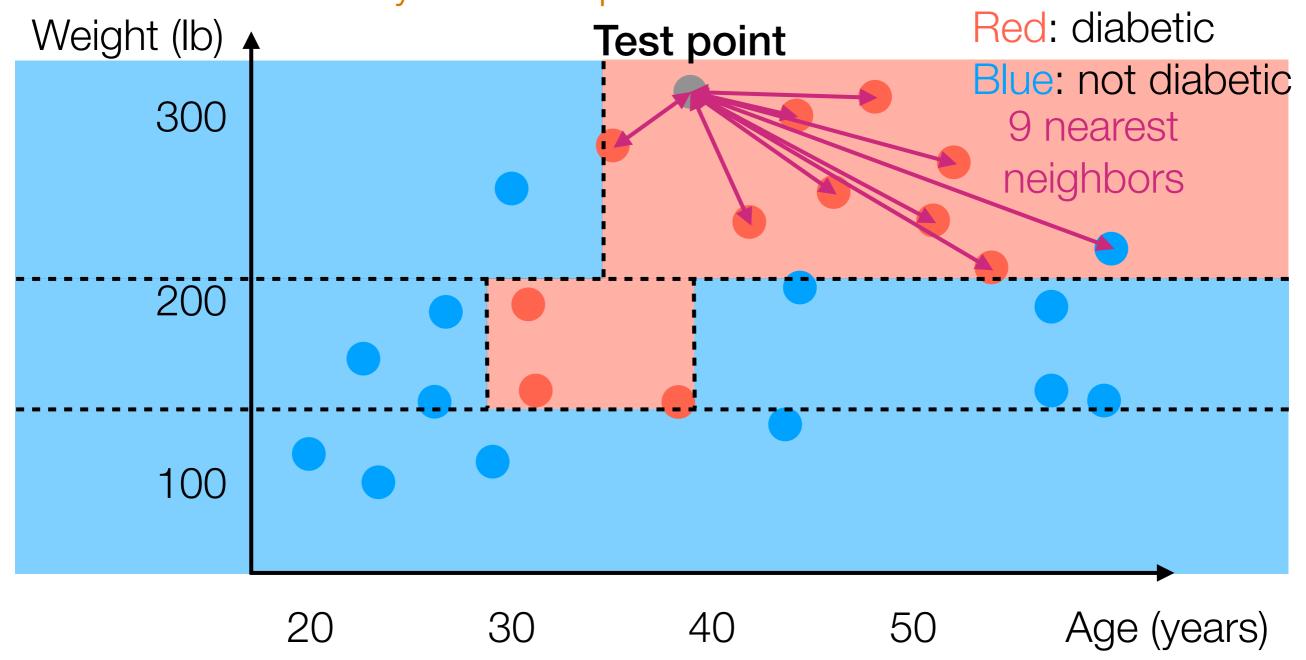
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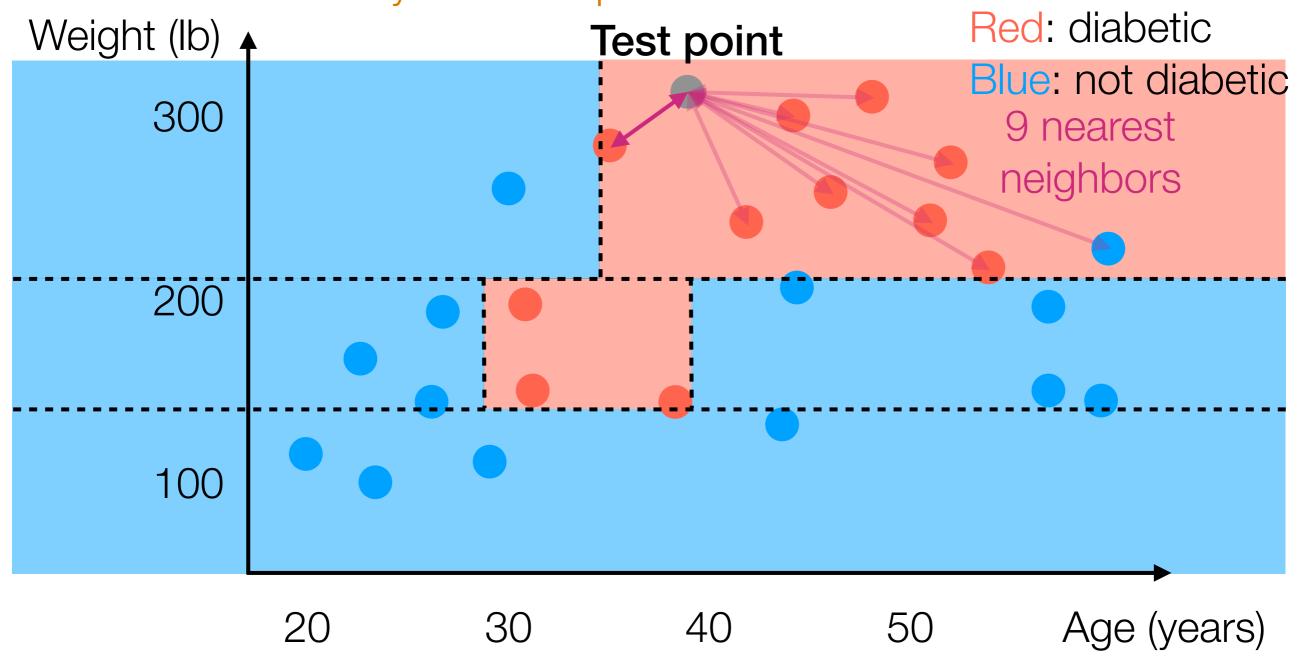
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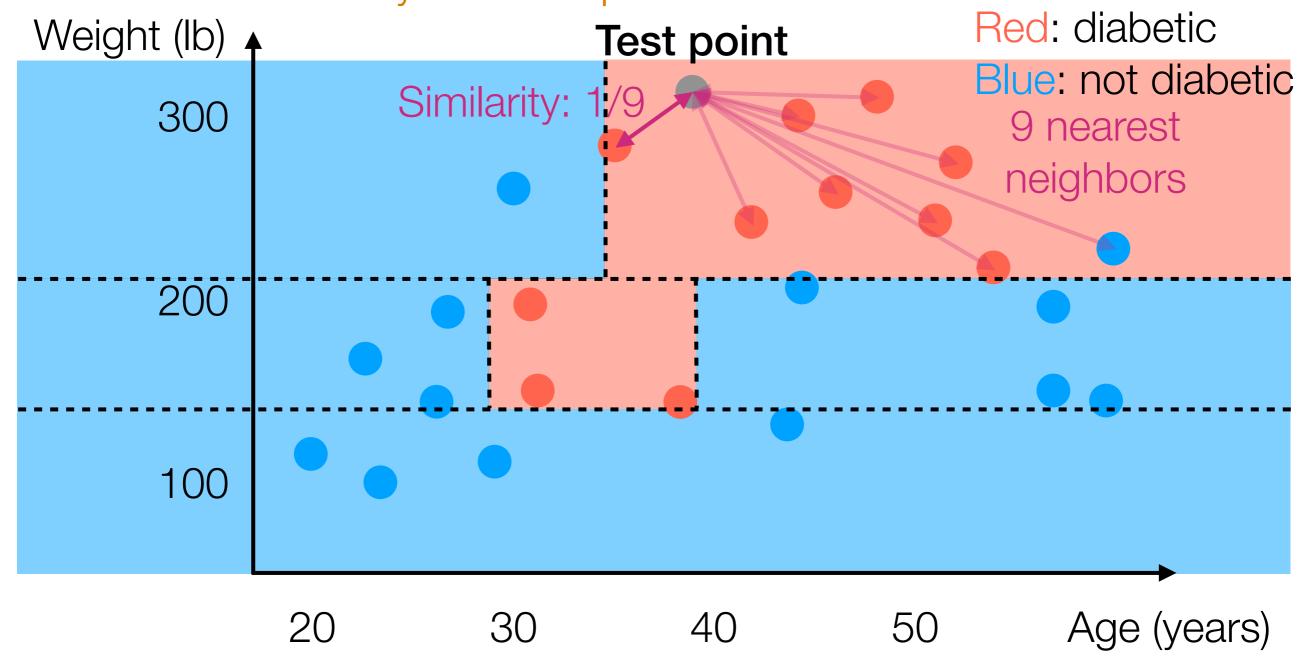
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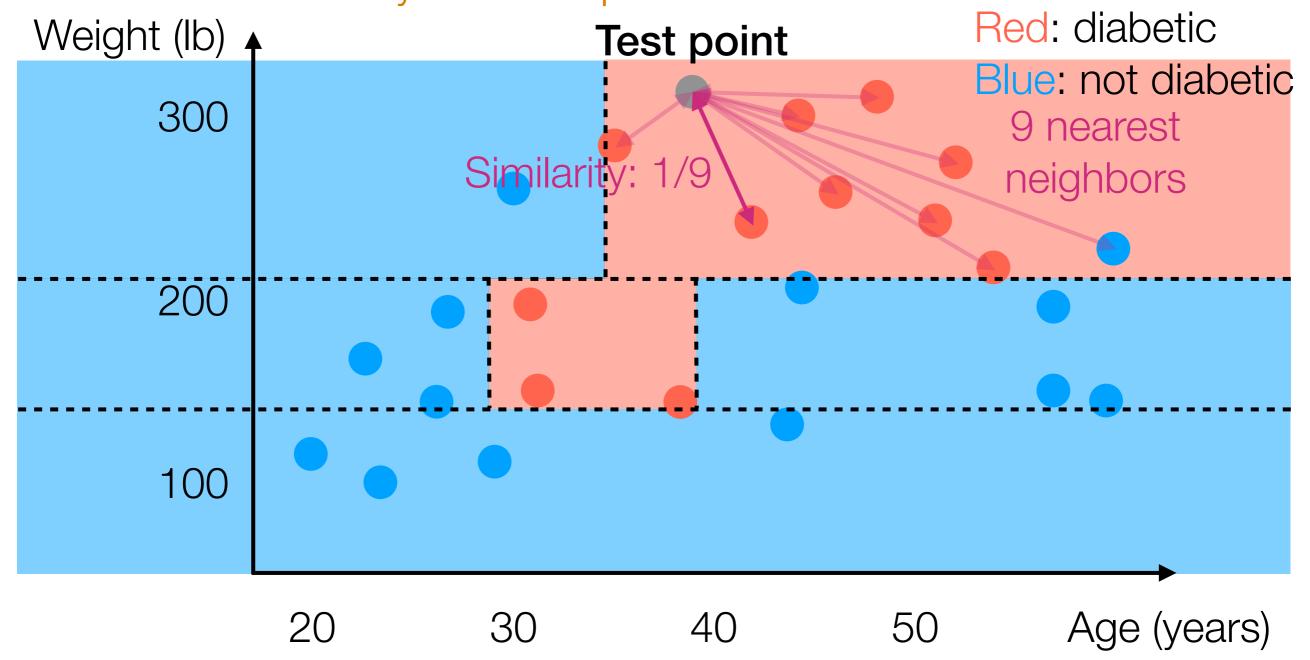
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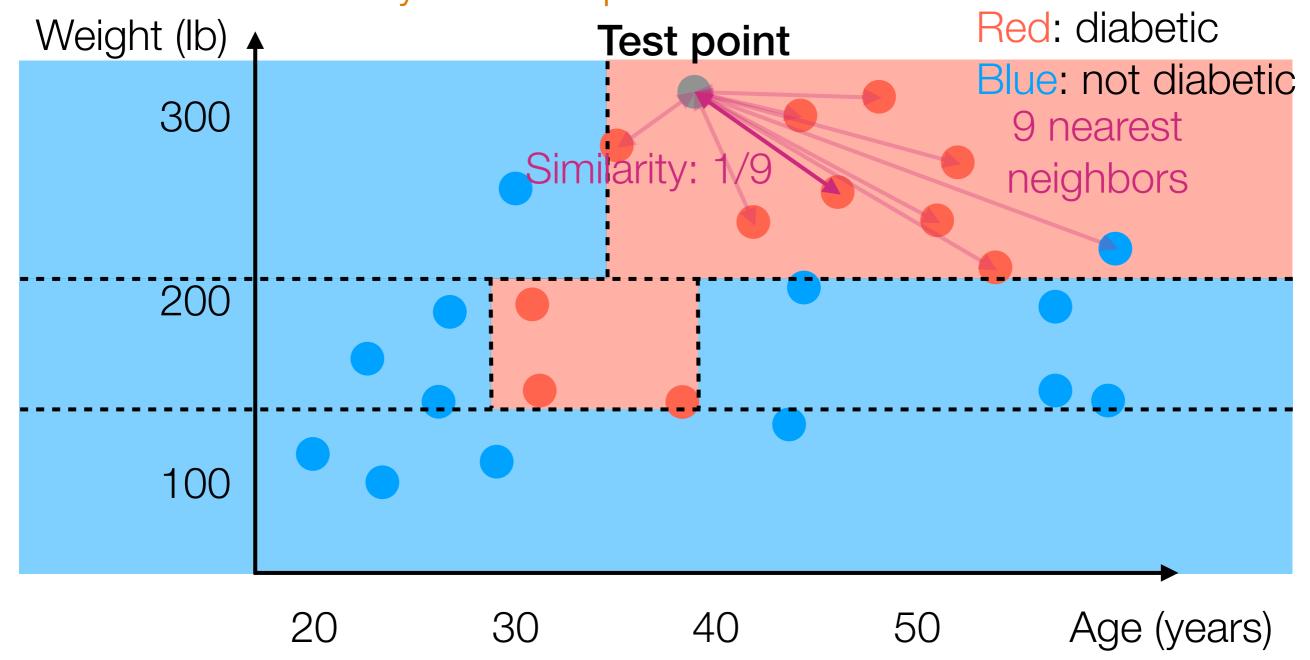
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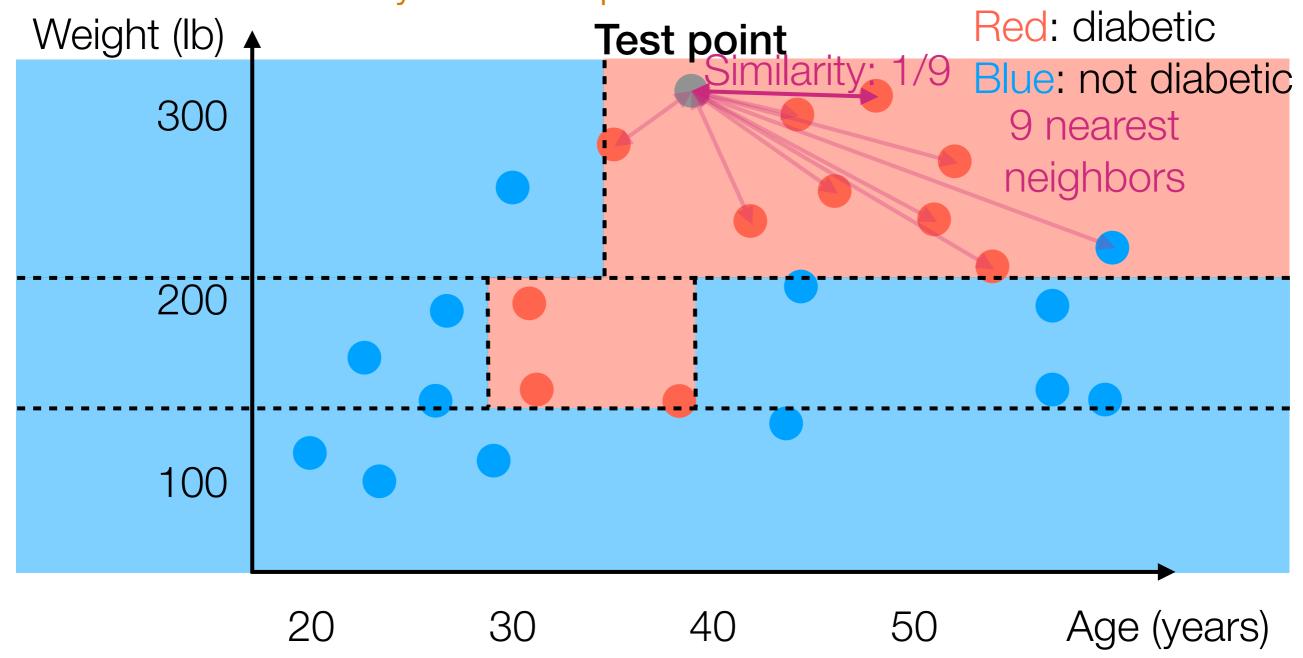
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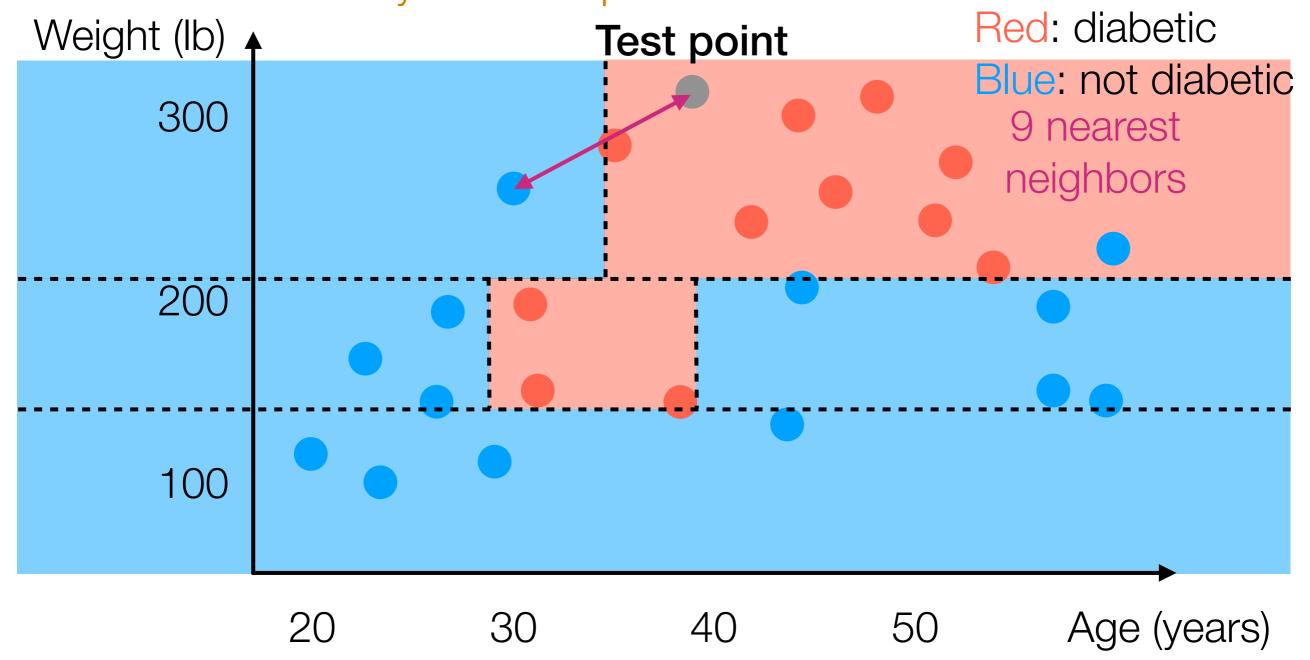
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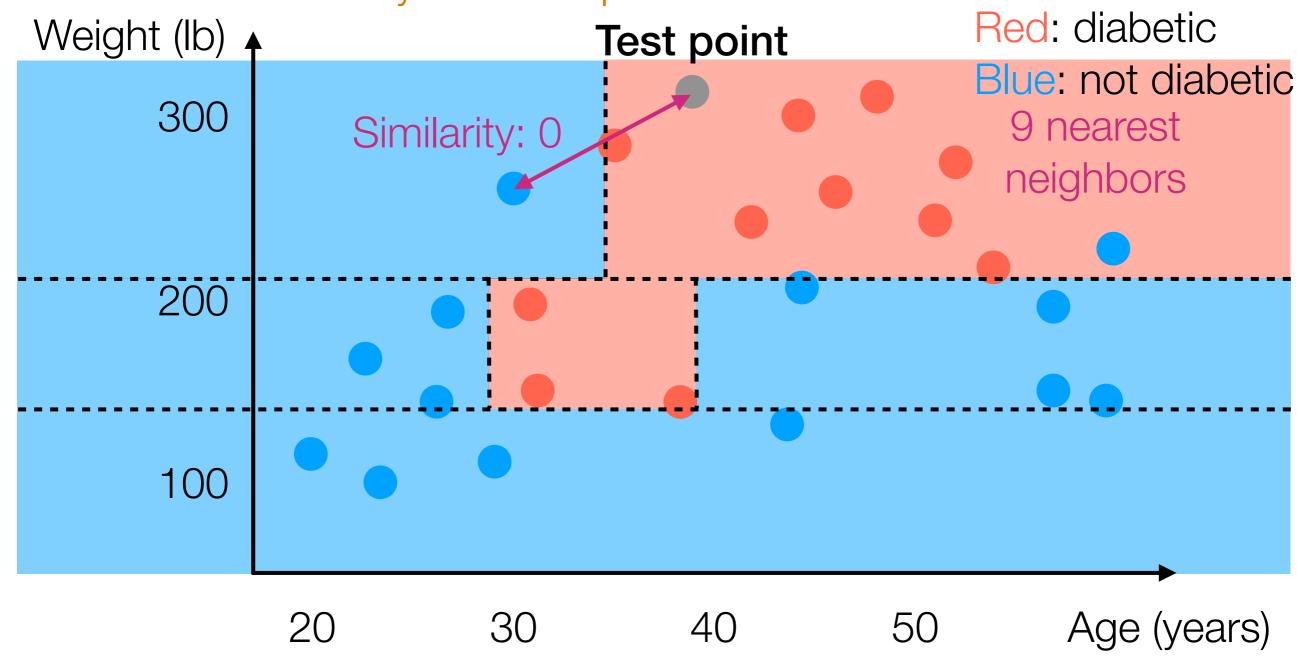
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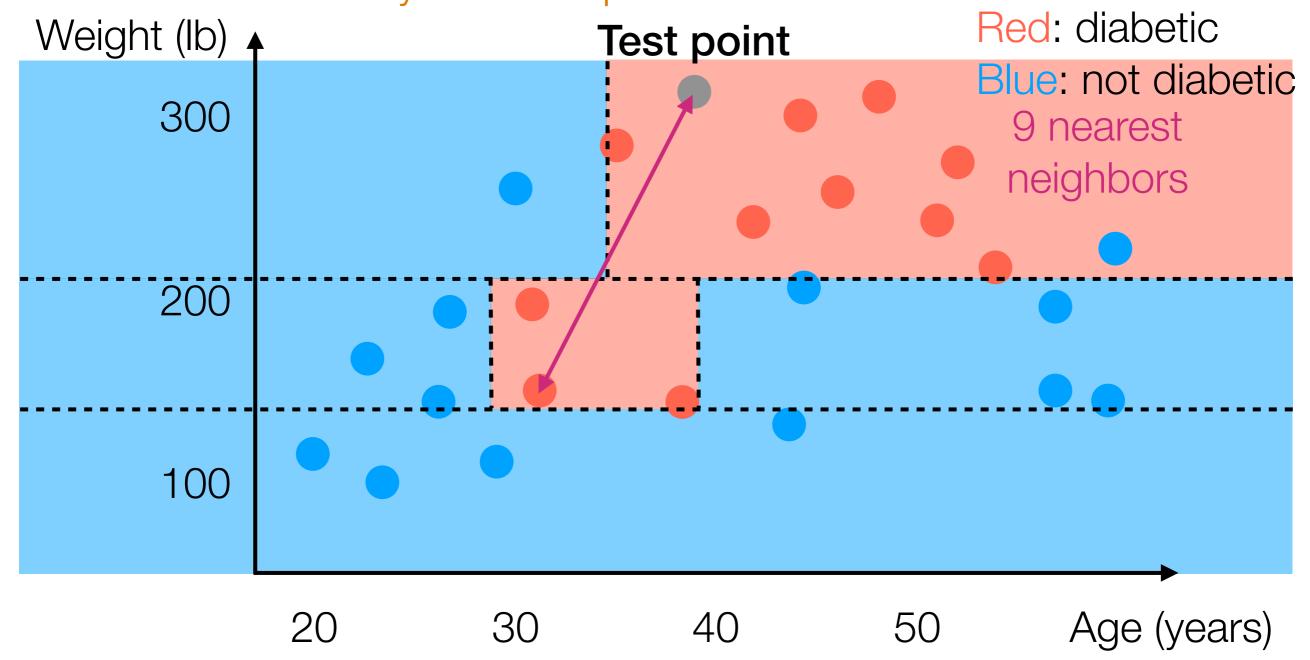
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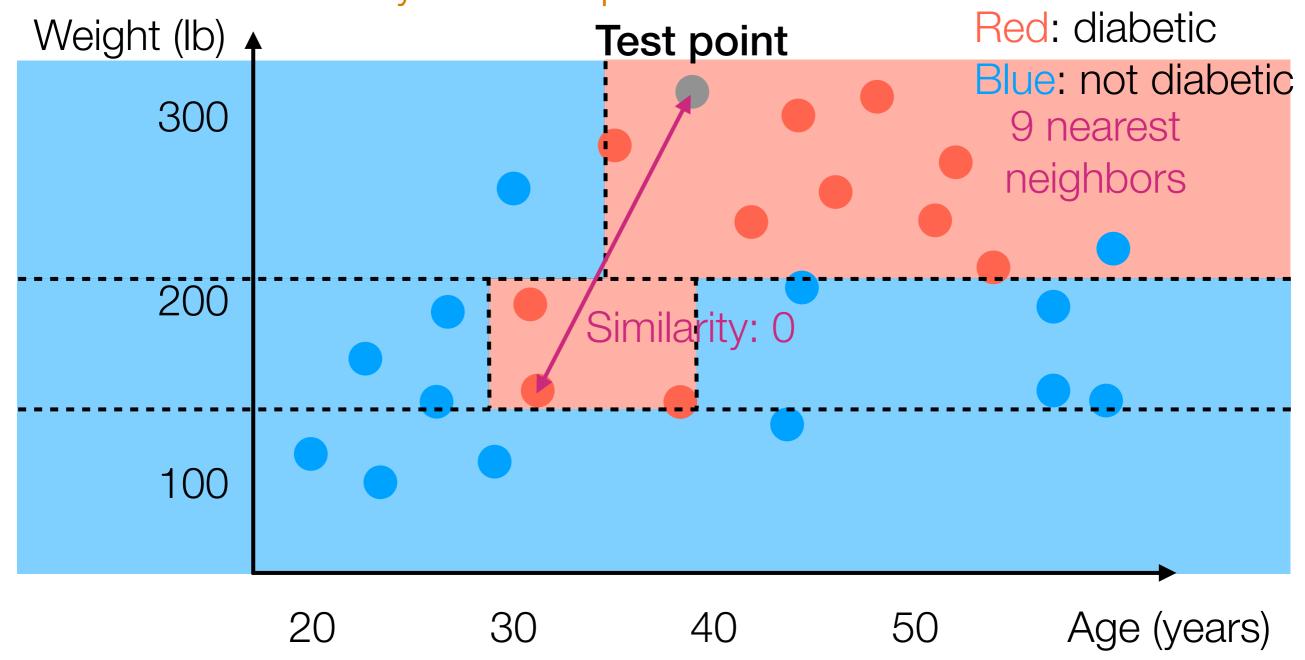
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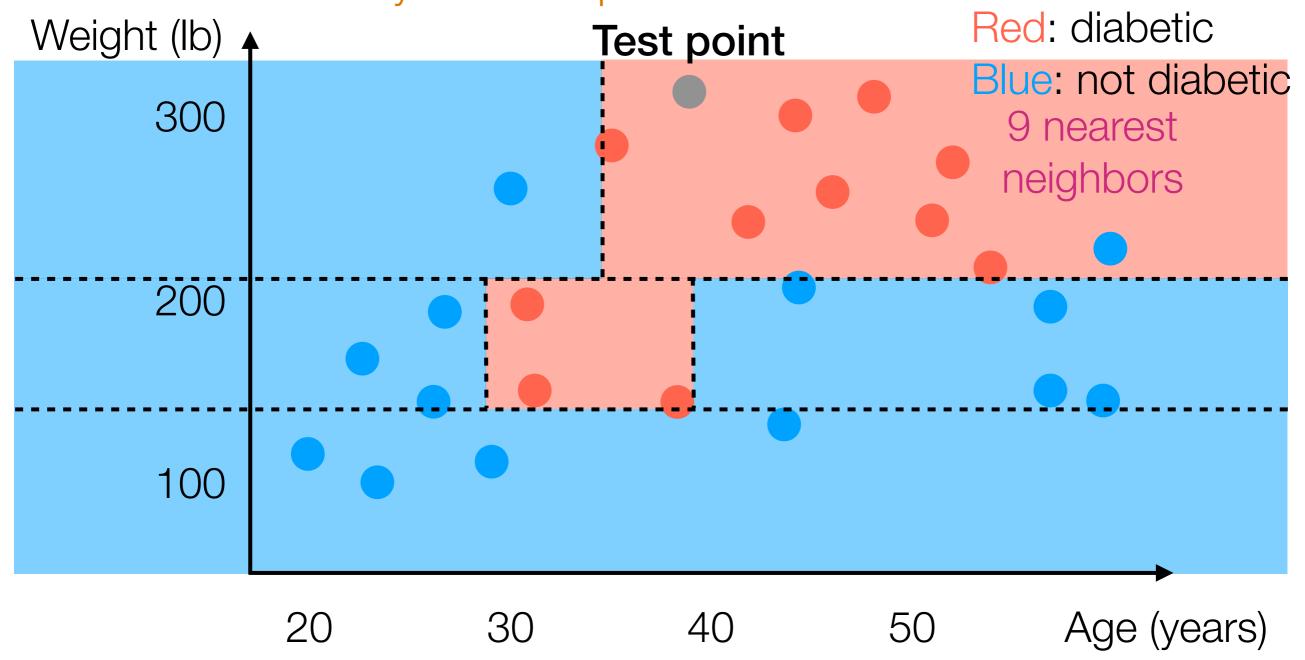
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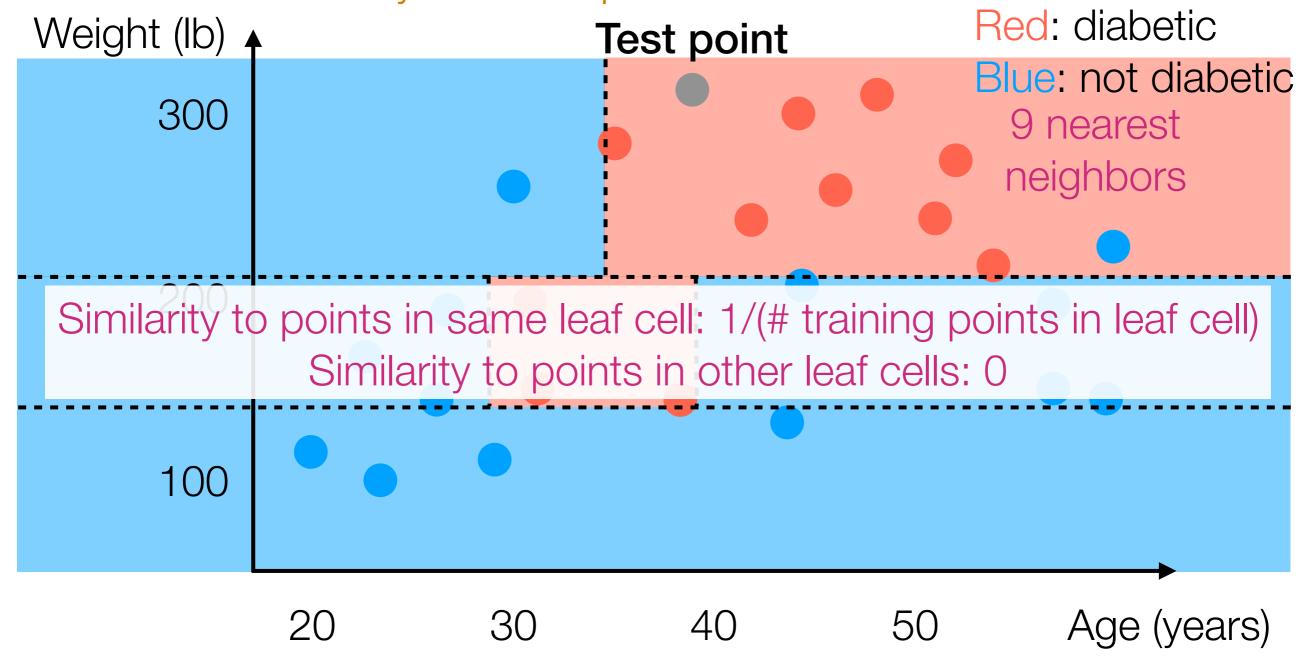
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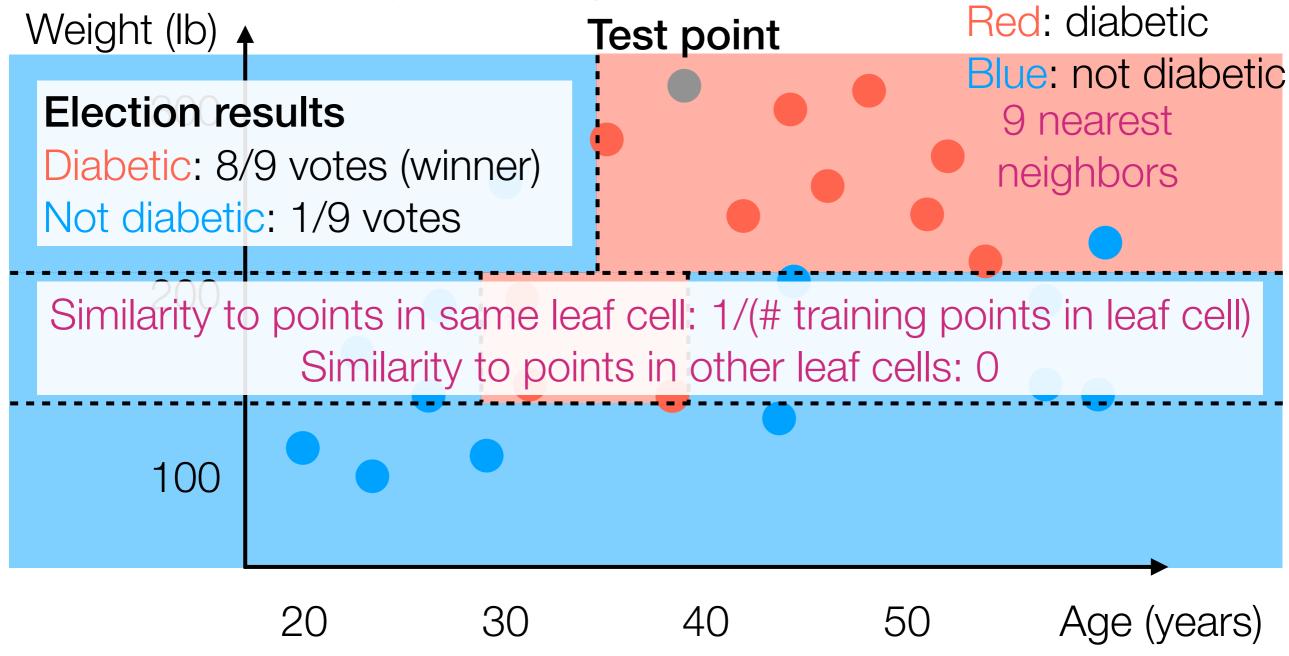
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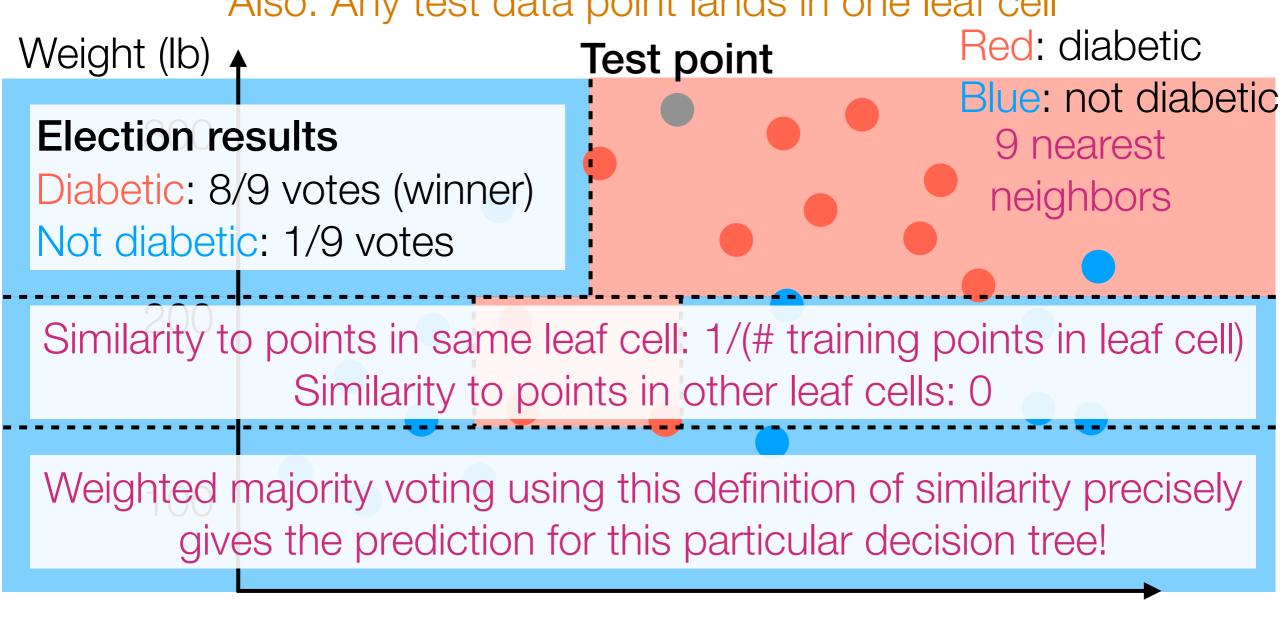
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Prediction for test point: majority vote of training points in same leaf cell (these training points act as nearest neighbors to the test point!)

40

50

Age (years)

30

20

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- Learning a decision tree learns a similarity function (that depends on labels)

Decision Tree for Classification Regression

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- Extremely easy to interpret and to do prediction
- Nearest neighbor interpretation:
 - For each test point, look at leaf cell it falls into to find its nearest neighbors among the training data (note: # of nearest neighbors varies!)
 - Prediction for test point: majority vote of nearest neighbors' labels
- Learning a decision tree learns a similarity function (that depends on labels)

 Typically, a decision tree is learned with randomness (e.g., we randomly chose which feature to threshold)

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Tree 1 Tree 2 Tree 3 \cdots Tree T

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Learn each tree separately using same training data

Tree 1

Tree 2

Tree 3

• • •

Tree *T*

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Learn each tree separately using same training data

New test data point

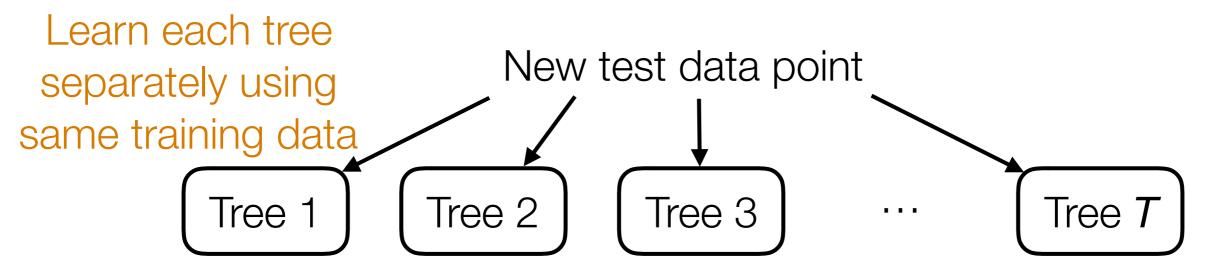
Tree 1

Tree 2

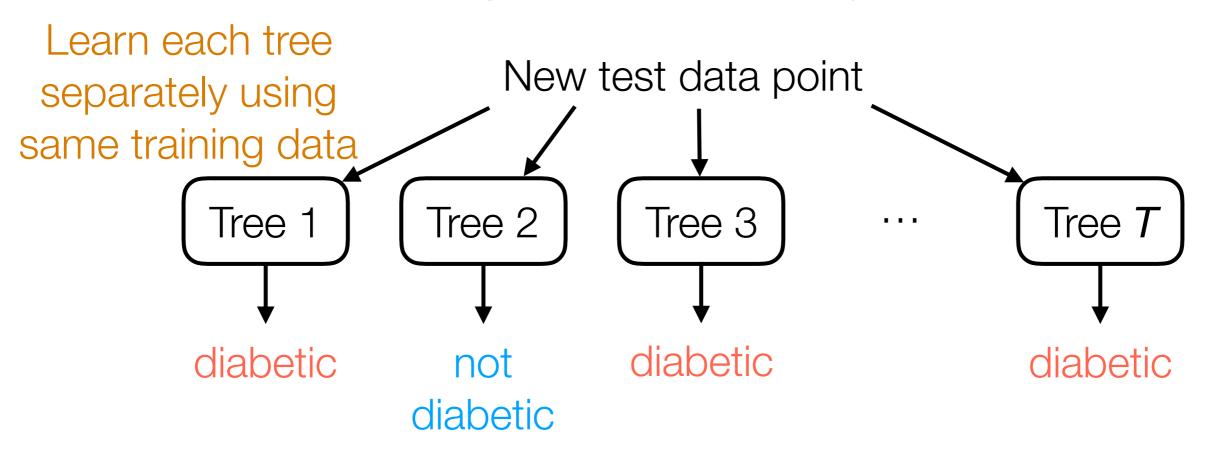
Tree 3

Tree T

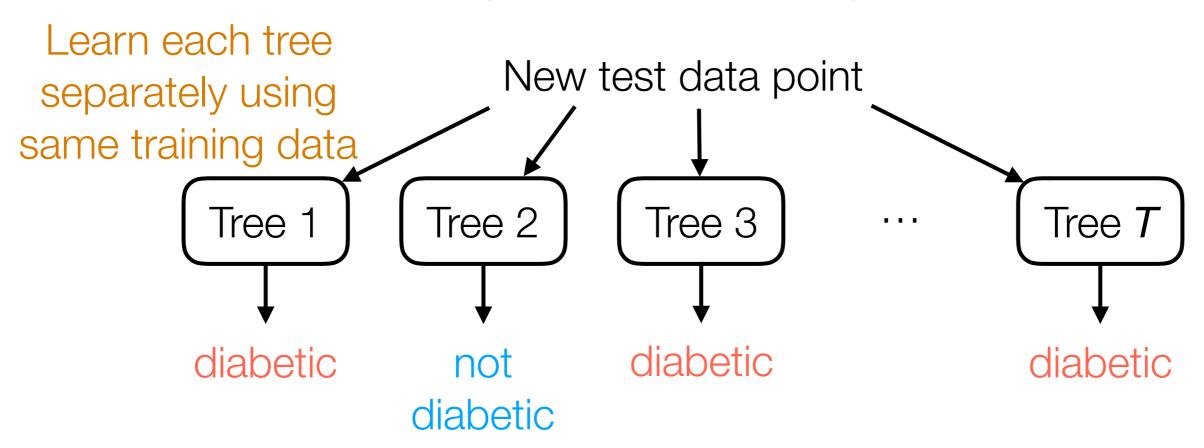
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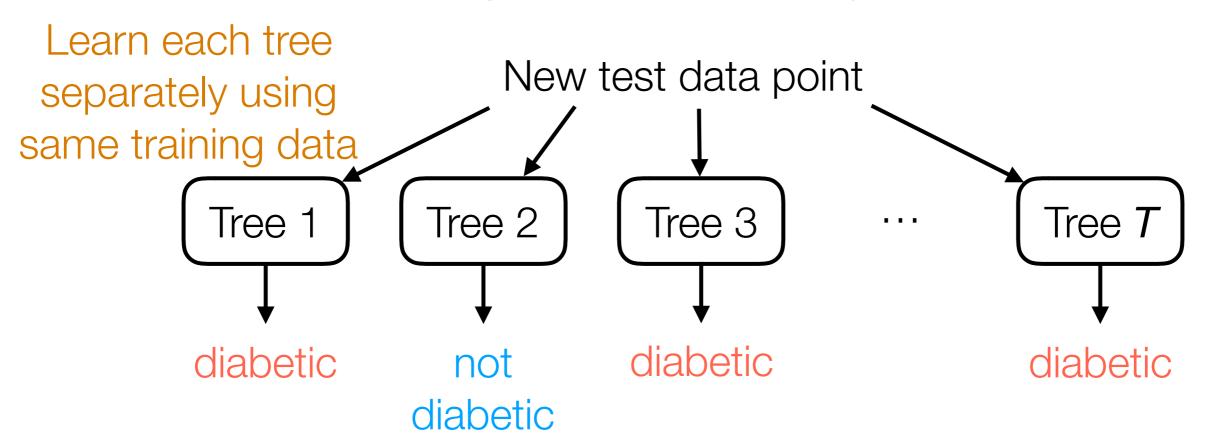


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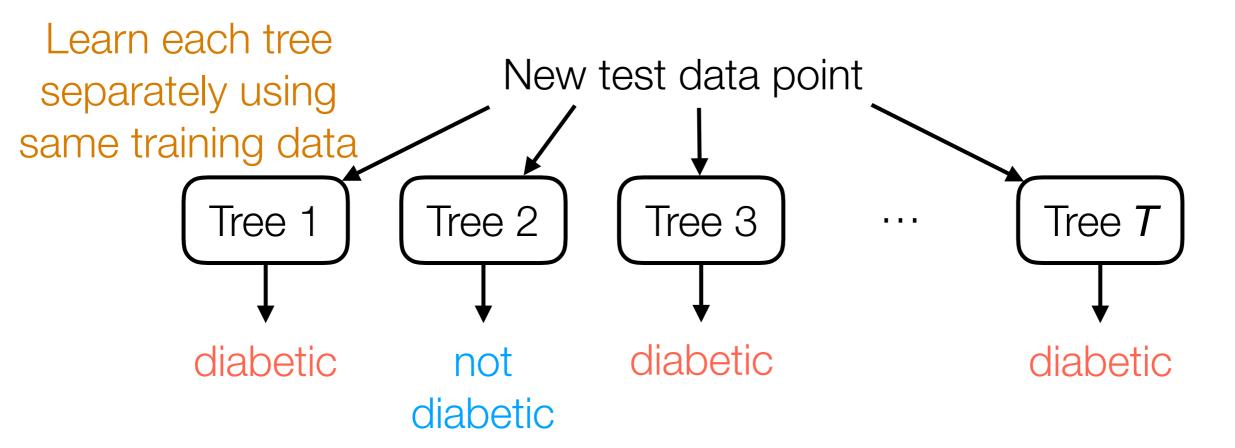


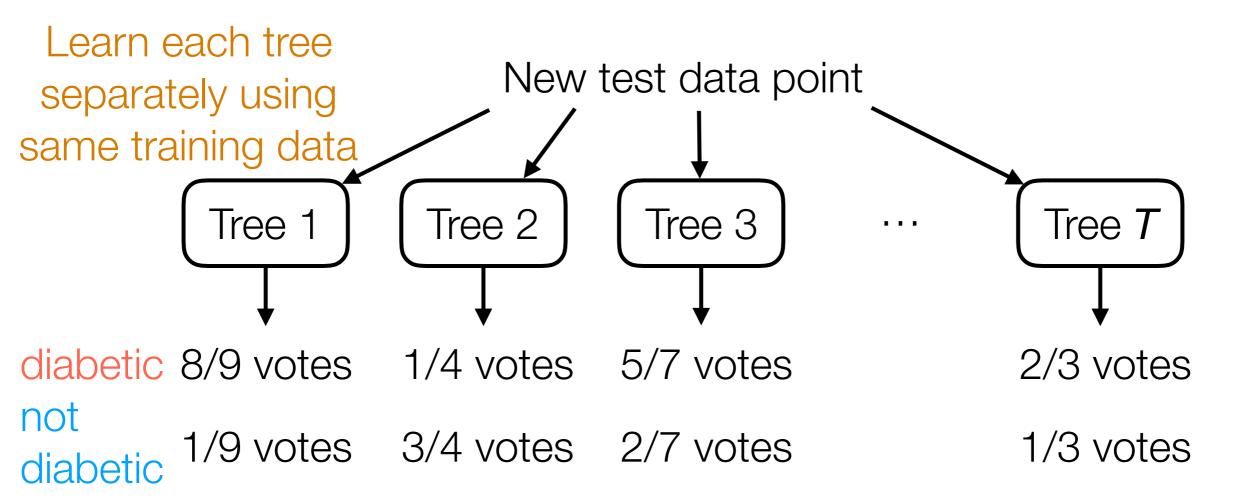
Final prediction: majority vote of the different trees' predictions

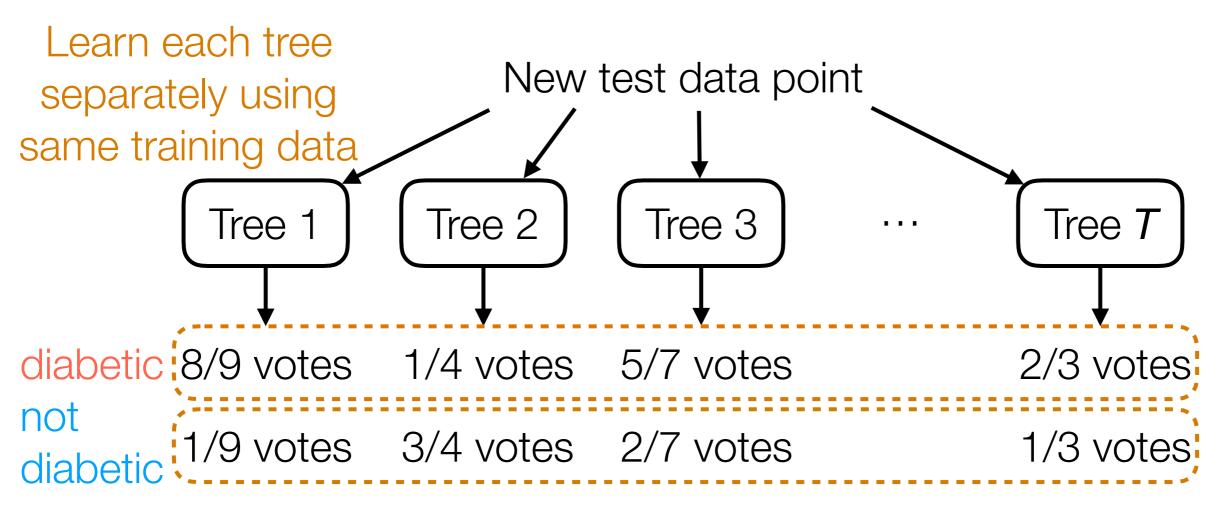
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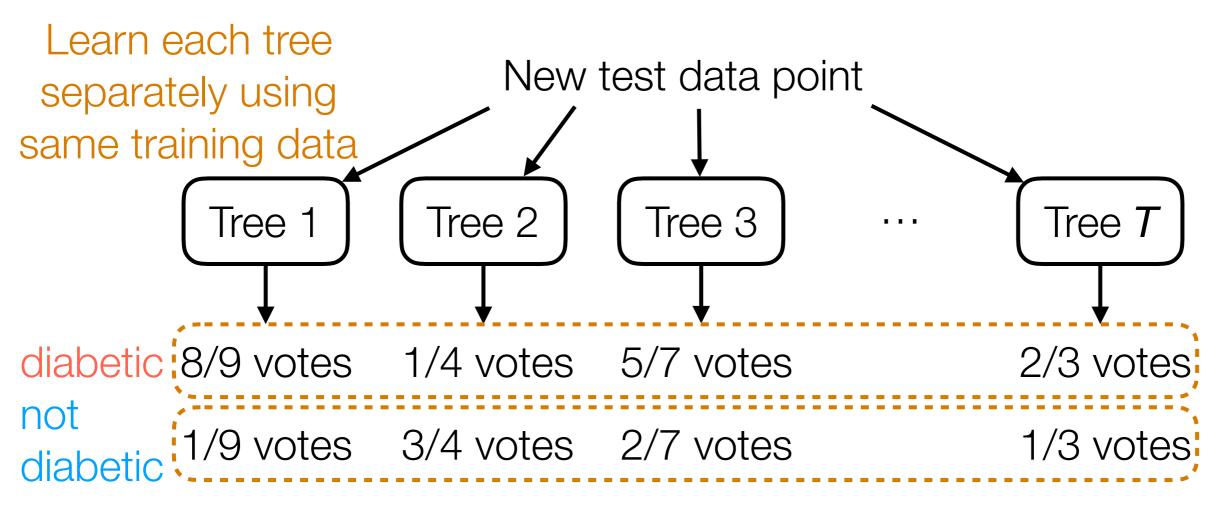
Final prediction: majority vote of the different trees' predictions. This is not the only way to aggregate predictions!





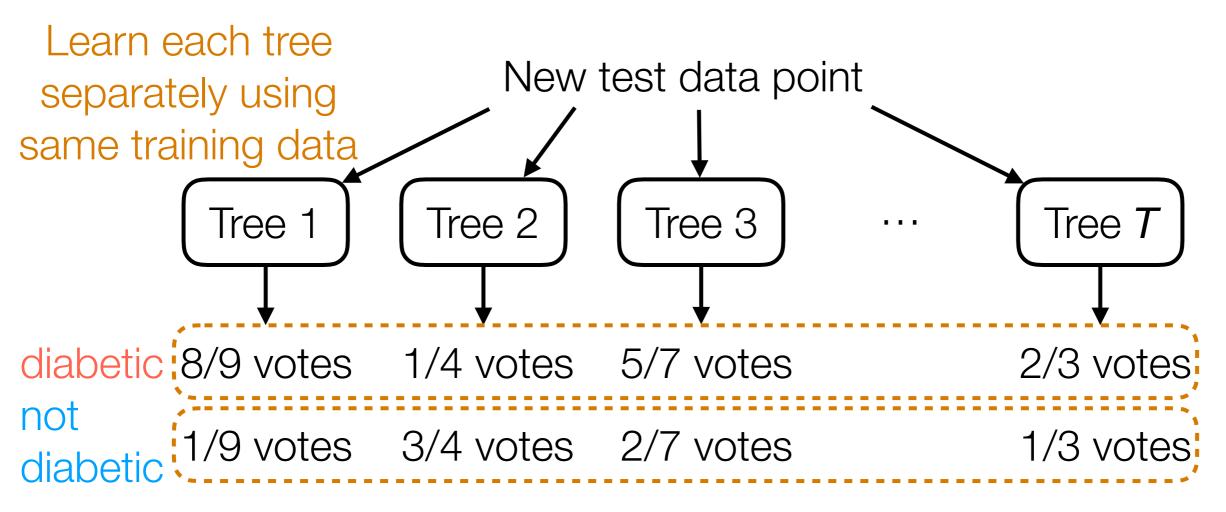


Final prediction: sum up votes across trees to find winner of election!



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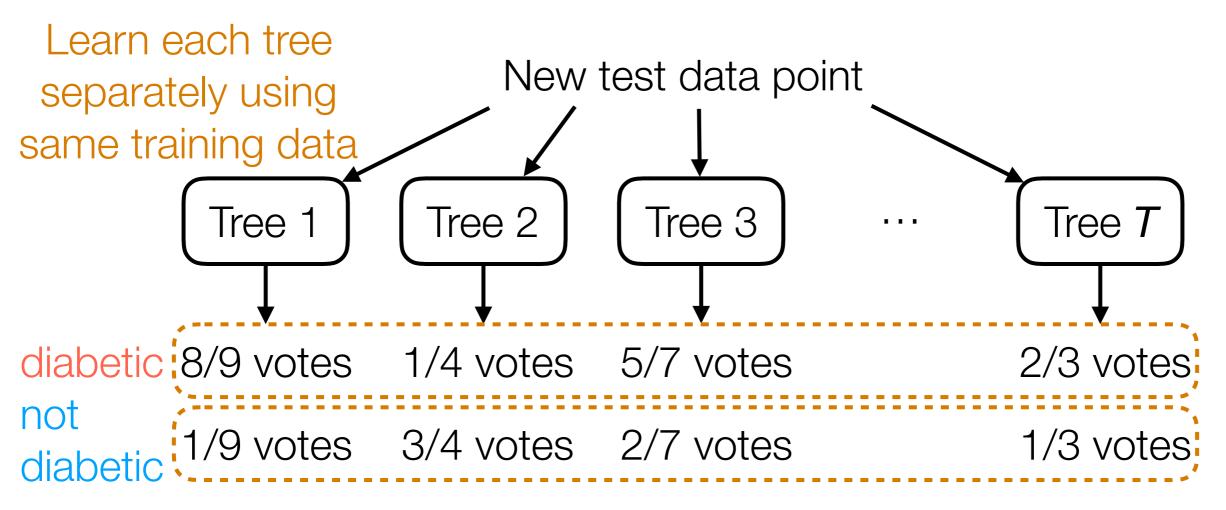
Nearest neighbor interpretation:



Final prediction: sum up votes across trees to find winner of election!

Nearest neighbor interpretation:

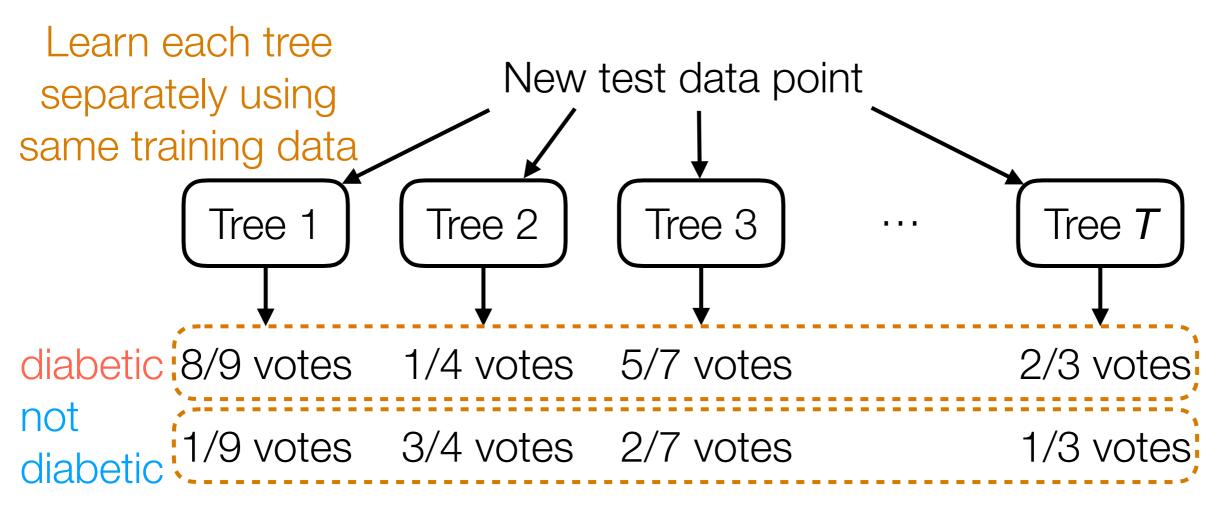
similarity
$$(x, x_i) = \frac{1}{T} \sum_{t=1}^{T} \text{similarity}_t(x, x_i)$$



Final prediction: sum up votes across trees to find winner of election!

Nearest neighbor interpretation:

similarity(
$$x, x_i$$
) = $\frac{1}{T} \sum_{t=1}^{T} \text{similarity}_t(x, x_i)$;
similarity function for t -th tree



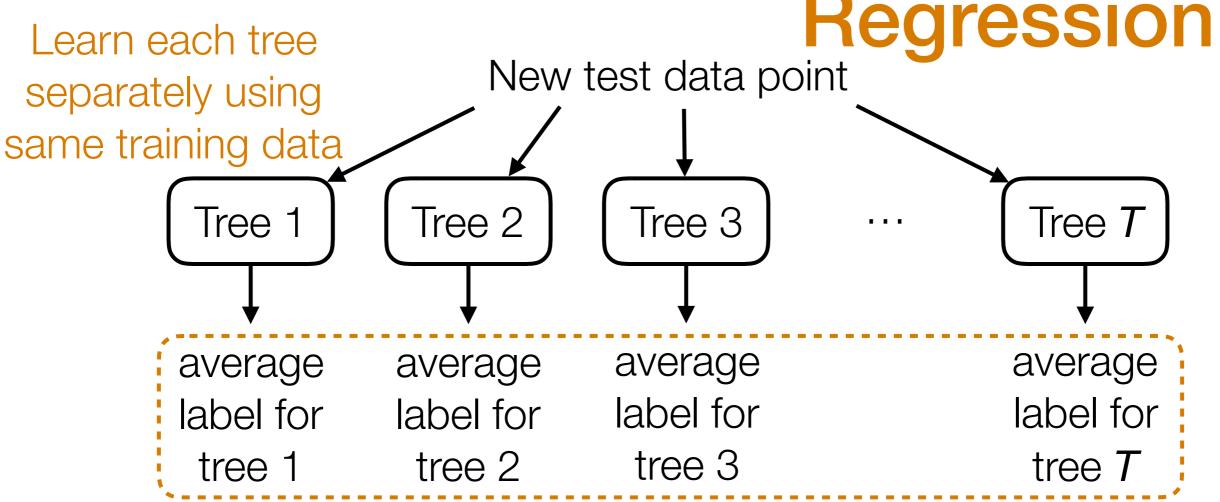
Final prediction: sum up votes across trees to find winner of election!

Nearest neighbor interpretation:

similarity(
$$x, x_i$$
) = $\frac{1}{T} \sum_{t=1}^{T} \text{similarity}_t(x, x_i)$;
makes overall similarity similarity function for t -th tree between 0 and 1

Decision Forest for Classification

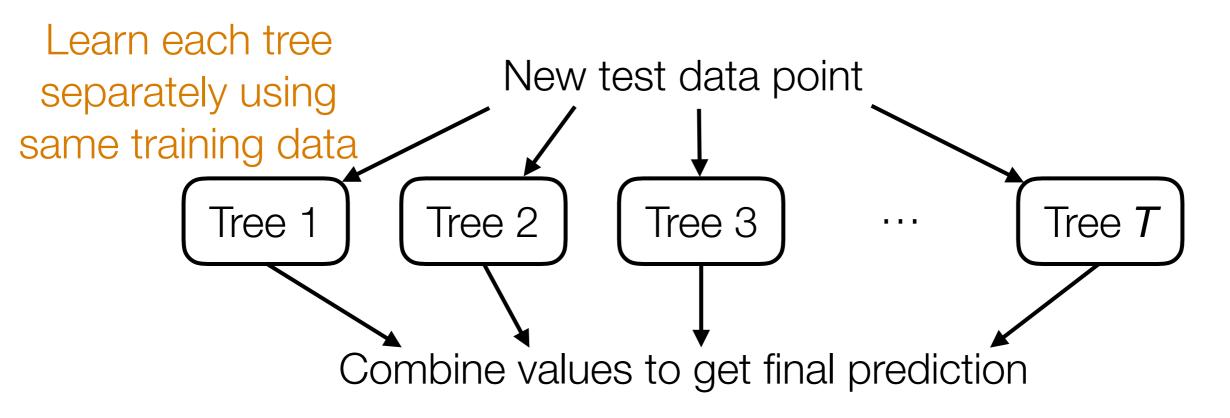
Regression

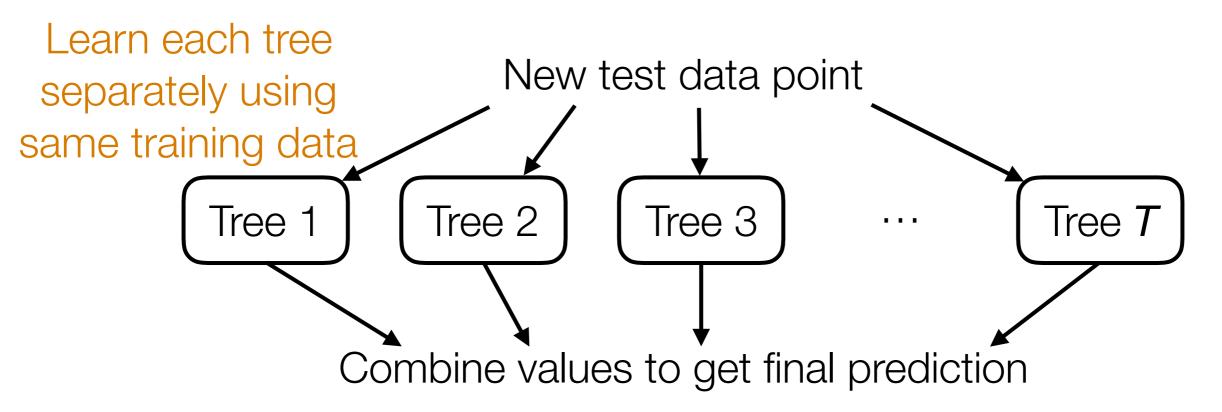


Average these values to get final prediction

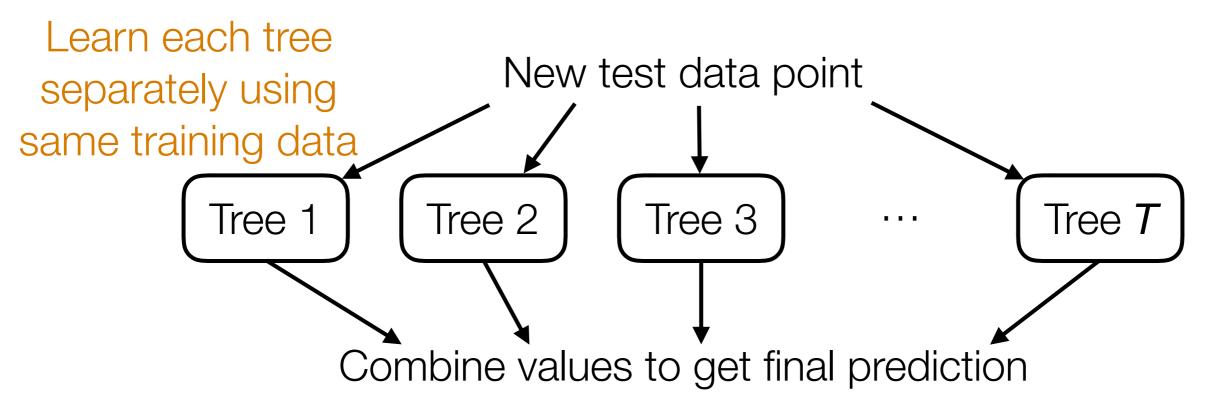
Nearest neighbor interpretation:

similarity(
$$x, x_i$$
) = $\sum_{t=1}^{T} \sum_{t=1}^{T} \text{similarity}_{t}(x, x_i)$;
makes overall similarity similarity function for t -th tree



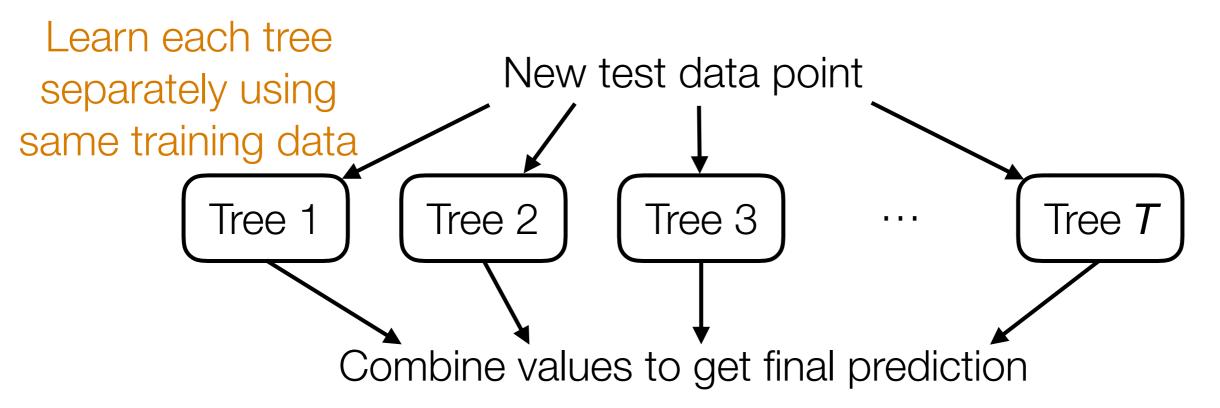


Question: What happens if all the trees are the same?



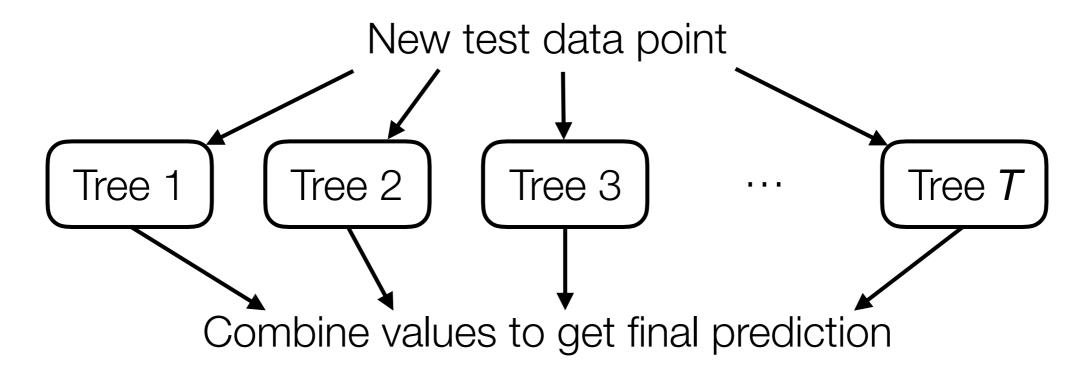
Question: What happens if all the trees are the same?

Adding randomness can make trees more different!



Question: What happens if all the trees are the same?

Adding randomness can make trees more different!



Question: What happens if all the trees are the same?

Adding randomness can make trees more different!

Decision Forest

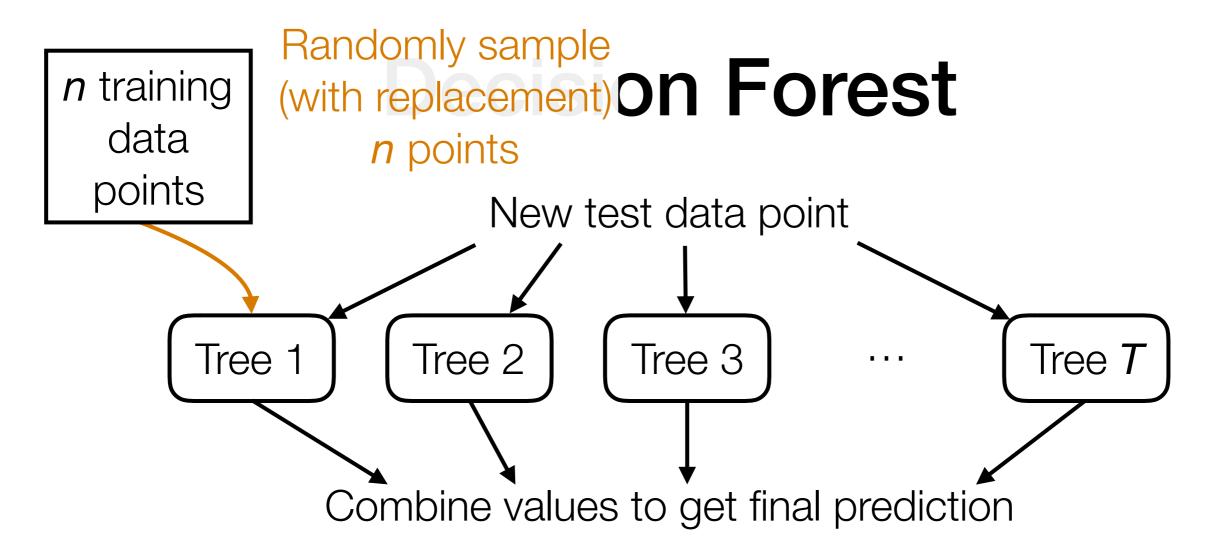
New test data point

Tree 1 Tree 2 Tree 3 ... Tree 7

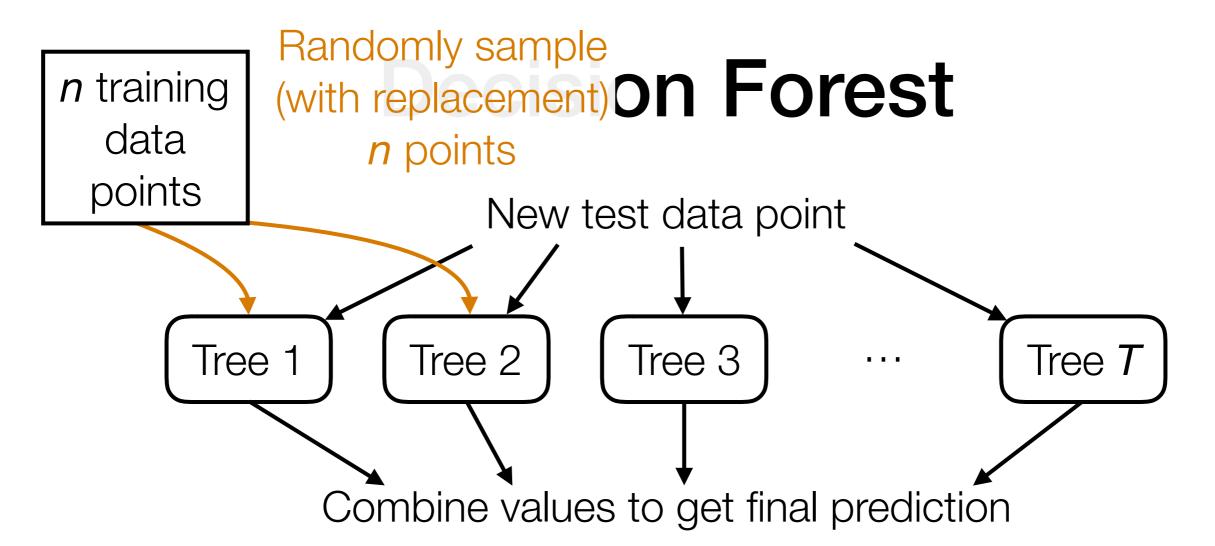
Combine values to get final prediction

Question: What happens if all the trees are the same?

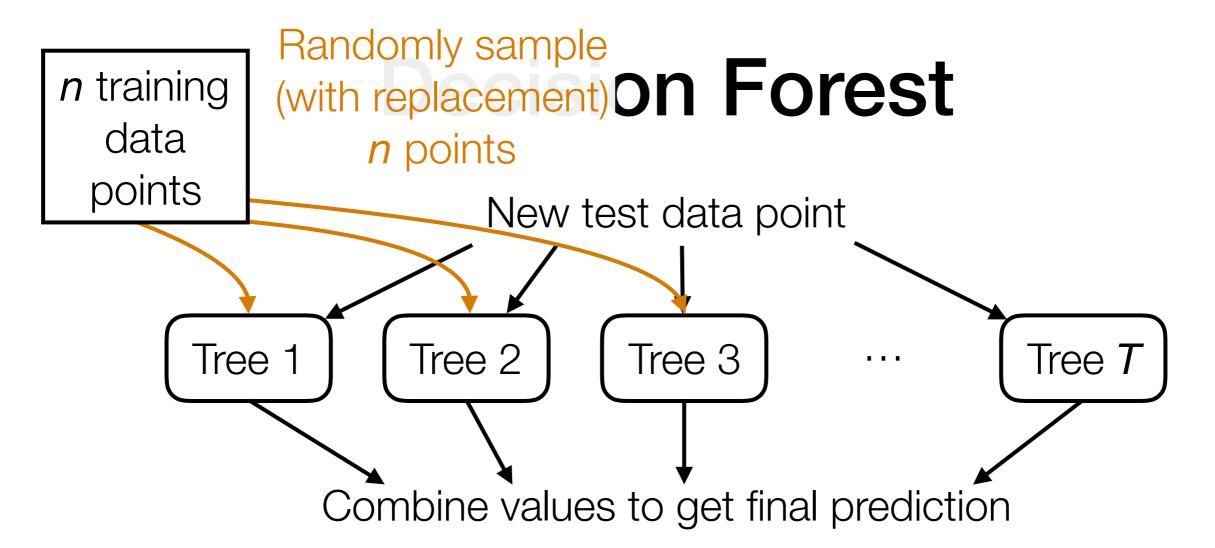
Adding randomness can make trees more different!



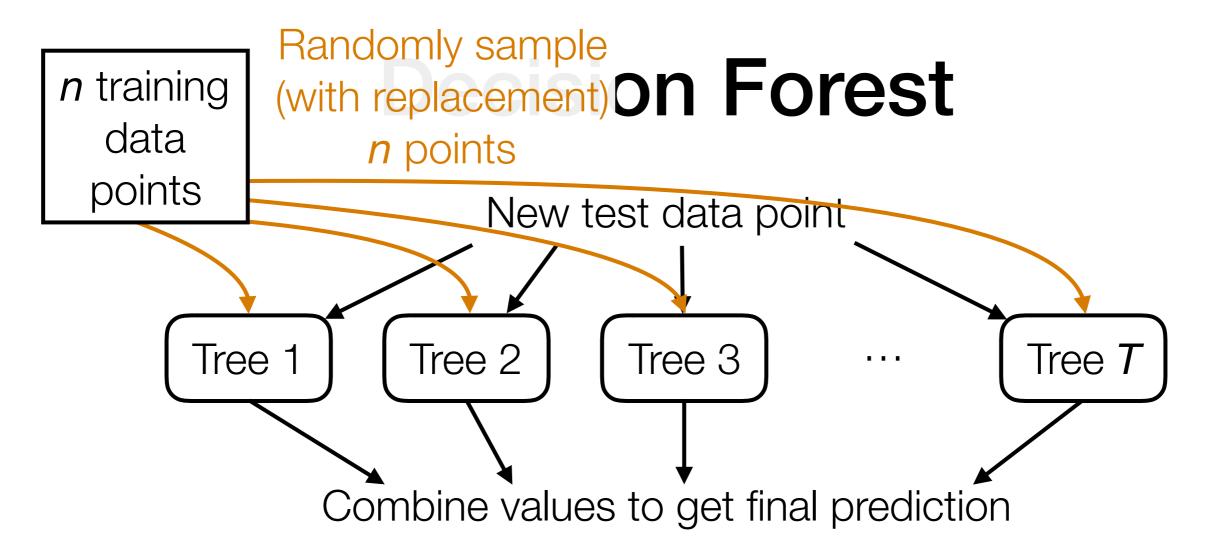
Adding randomness can make trees more different!



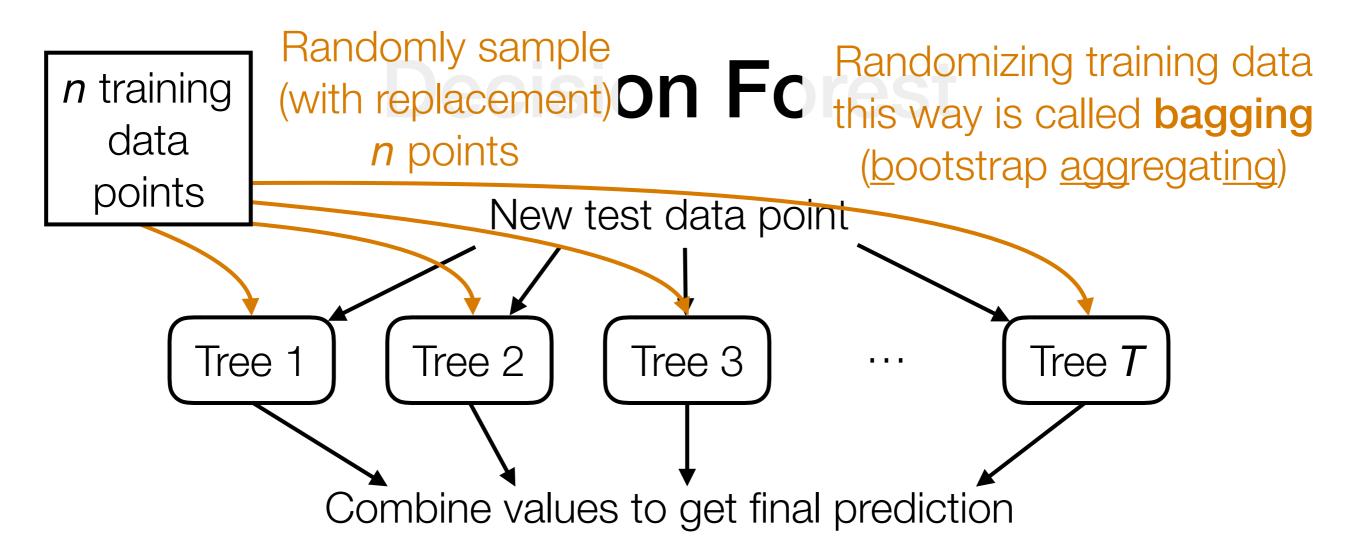
Adding randomness can make trees more different!



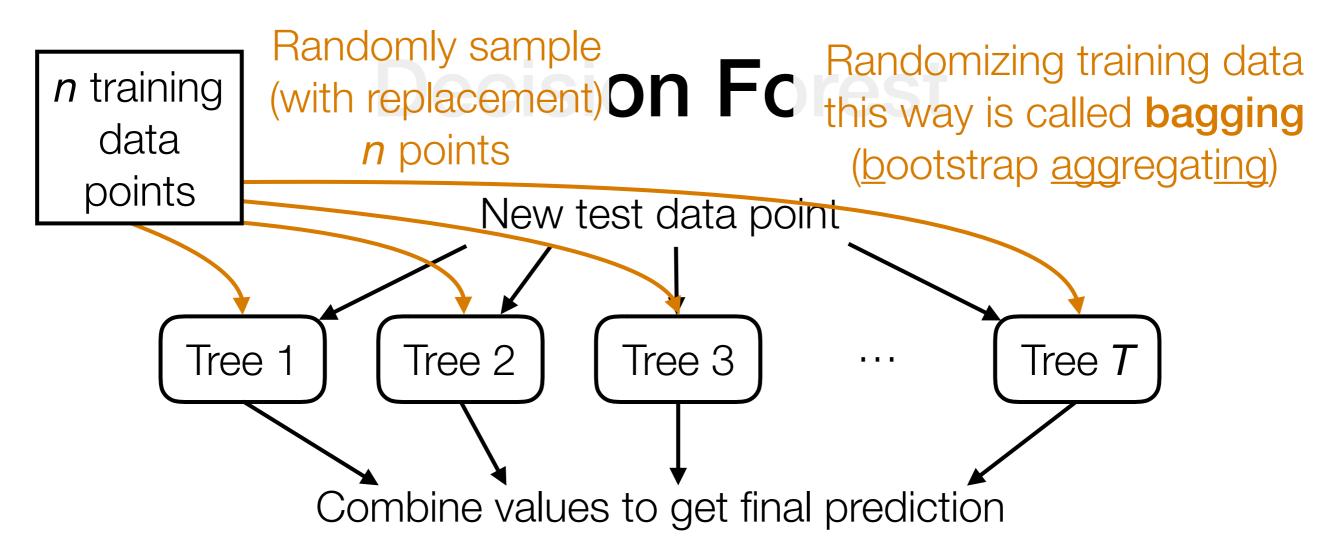
Adding randomness can make trees more different!



Adding randomness can make trees more different!



Adding randomness can make trees more different!



Adding randomness can make trees more different!

- Random Forest: in addition to randomly choosing features to threshold, also randomize training data used for each tree
- Extremely randomized trees: further randomize thresholds rather than trying to pick clever thresholds

I'll only sketch the general idea

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Random decision forests learned each tree separately

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Random decision forests learned each tree separately

If some trees are bad, we still weight them equally

I'll only sketch the general idea

Random decision forests learned each tree separately

Boosting: learn trees *sequentially*, and learn from previous trees' mistakes

If some trees are bad, we still weight them equally

I'll only sketch the general idea

Random decision forests learned each tree separately

Boosting: learn trees *sequentially*, and learn from previous trees' mistakes

If some trees are bad, we still weight them equally

Boosting: weight trees unequally so bad trees are down-weighted

Tree 1

Training data







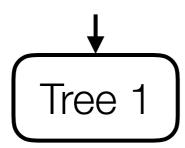
Tree 1

Training data







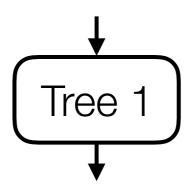


Training data







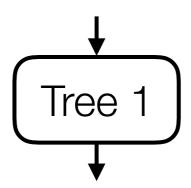


Training data









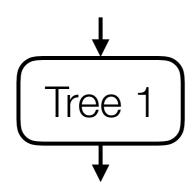
Predicted: cat, dog, shark

Training data









Predicted: cat, dog, shark

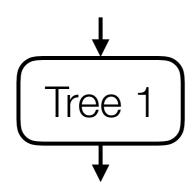
Actual: cat, cat, robot

Training data









Predicted: cat, dog, shark

Actual: cat, cat, robot

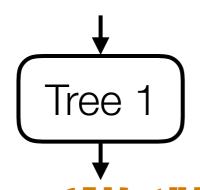
Where did the errors appear?

Training data









Predicted: cat, dog; shark

Actual: cat, cat, robot

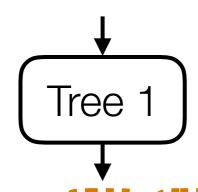
Where did the errors appear?

Training data









Predicted: cat, dog; shark

Actual: cat, cat, robot

Where did the errors appear?

Training data









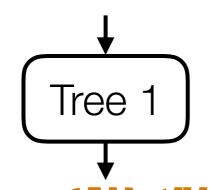












Predicted: cat, dog; shark

Actual: cat, cat, robot

Where did the errors appear?

Training data



Training data







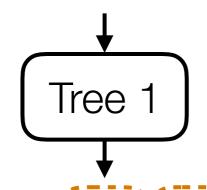












Predicted: cat, dog; shark

Actual: cat, cat, robot

Tree 2

Where did the errors appear?

Training data







Training data

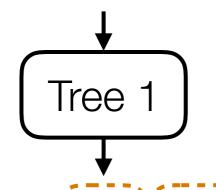








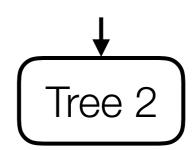




Predicted: cat, dog; shark

Actual: cat, cat, robot

Where did the errors appear?



Training data









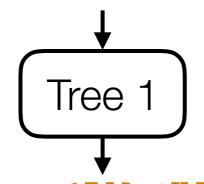












Predicted: cat, dog; shark

Actual: cat, cat, robot

Tree 2

Where did the errors appear?

Training data







Training data

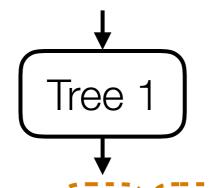












Predicted: cat, dog; shark

Actual: cat, cat, robot

Tree 2

Predicted: cat, cat, donkey

Actual: cat, cat, robot

Where did the errors appear?

Training data







Training data

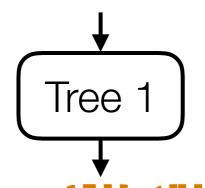












Predicted: cat, dog, shark

Actual: cat, cat, robot

Tree 2

Predicted: cat, cat, donkey

Actual: cat, cat, robot

Where did the errors appear?

Where did the errors appear?

Training data







Training data

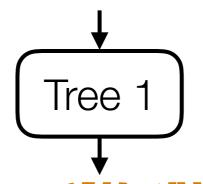












Predicted: cat, dog, shark

Actual: cat, cat, robot

Tree 2

Predicted: cat, cat, donkey

Actual: cat, cat, robot

Where did the errors appear?

Duplicate these training examples to emphasize them more when learning the next tree

Where did the errors appear?

Training data







Training data

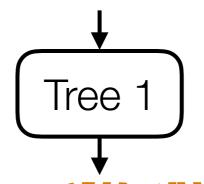












Predicted: cat, dog; shark

Actual: cat, cat, robot

Tree 2

Predicted: cat, cat, donkey

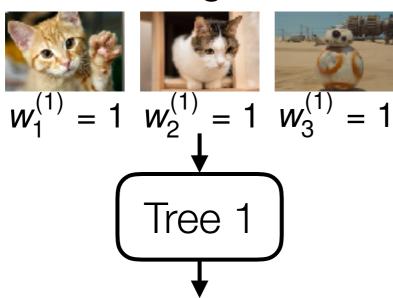
Actual: cat, cat, robot

Where did the errors appear?

Duplicate these training examples to emphasize them more when learning the next tree

Where did the errors appear?

Training data



Predicted: cat, dog; shark

Actual: cat, cat, robot

Where did the errors appear?

Duplicate these training examples to emphasize them more when learning the next tree

Training data

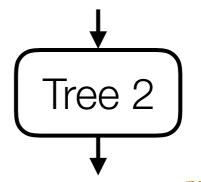










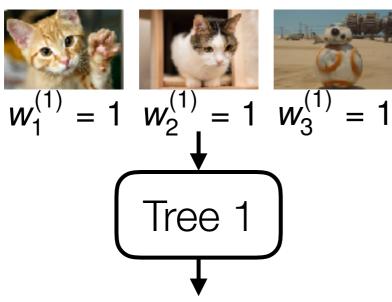


Predicted: cat, cat, donkey

Actual: cat, cat, robot

Where did the errors appear?

Training data



Predicted: cat, dog; shark

Actual: cat, cat, robot

Where did the errors appear?

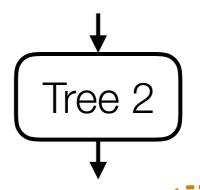
Duplicate these training examples to emphasize them more when learning the next tree

Training data







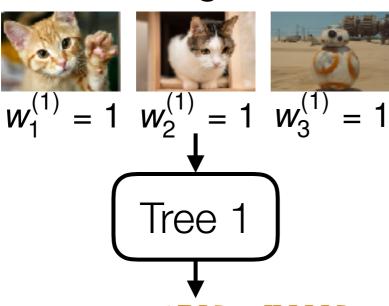


Predicted: cat, cat, donkey

Actual: cat, cat, robot

Where did the errors appear?

Training data



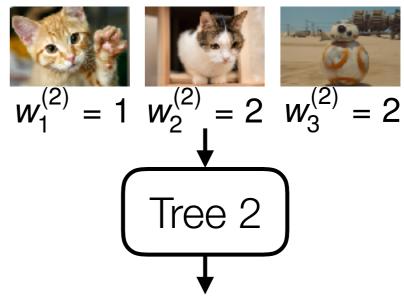
Predicted: cat, dog, shark

Actual: cat, cat, robot

Where did the errors appear?

Duplicate these training examples to emphasize them more when learning the next tree

Training data



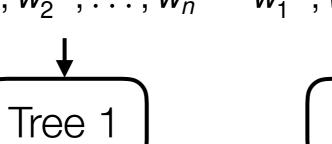
Predicted: cat, cat, donkey

Actual: cat, cat, robot

Where did the errors appear?

Training data: x_1, x_2, \ldots, x_n x_1, x_2, \ldots, x_n

Weights: $w_1^{(1)}, w_2^{(1)}, \dots, w_n^{(1)}$ $w_1^{(2)}, w_2^{(2)}, \dots, w_n^{(2)}$

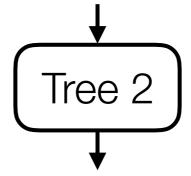


Predicted: $\widehat{y}_1^{(1)}, \widehat{y}_2^{(1)}, \dots, \widehat{y}_n^{(1)} \qquad \widehat{y}_1^{(2)}, \widehat{y}_2^{(2)}, \dots, \widehat{y}_n^{(2)}$

Actual: y_1, y_2, \ldots, y_n

$$X_1, X_2, \ldots, X_n$$

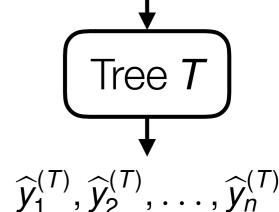
 $W_1^{(2)}, W_2^{(2)}, \ldots, W_n^{(2)}$



 y_1, y_2, \ldots, y_n

$$X_1, X_2, \dots, X_n$$

 $W_1^{(T)}, W_2^{(T)}, \dots, W_n^{(T)}$



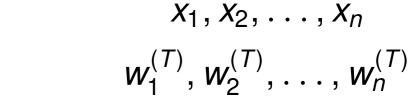
$$y_1, y_2, \ldots, y_n$$

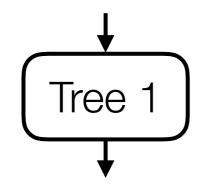
Learn trees sequentially accounting for mistakes made previously

Training data: x_1, x_2, \ldots, x_n x_1, x_2, \ldots, x_n

$$X_1, X_2, \ldots, X_n$$

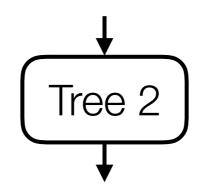
Weights: $w_1^{(1)}, w_2^{(1)}, \dots, w_n^{(1)}, w_1^{(2)}, w_2^{(2)}, \dots, w_n^{(2)}$





Predicted: $\widehat{y}_{1}^{(1)}, \widehat{y}_{2}^{(1)}, \dots, \widehat{y}_{n}^{(1)}$ $\widehat{y}_{1}^{(2)}, \widehat{y}_{2}^{(2)}, \dots, \widehat{y}_{n}^{(2)}$

Actual: $y_1, y_2, ..., y_n$ $y_1, y_2, ..., y_n$



Tree
$$T$$

$$\widehat{y}_1^{(T)}, \widehat{y}_2^{(T)}, \dots, \widehat{y}_n^{(T)}$$

 y_1, y_2, \ldots, y_n

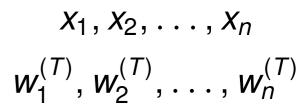
Learn trees sequentially accounting for mistakes made previously

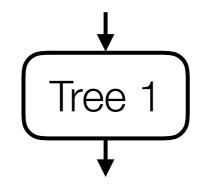


Weights:
$$w_1^{(1)}, w_2^{(1)}, \dots, w_n^{(1)}, w_1^{(2)}, w_2^{(2)}, \dots, w_n^{(2)}, \dots, w_n^{(2)}$$
 $w_1^{(T)}, w_2^{(T)}, \dots, w_n^{(T)}$

$$X_1, X_2, \ldots, X_n$$

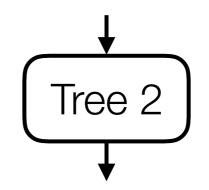
$$W_1^{(2)}, W_2^{(2)}, \ldots, W_n^{(2)}$$





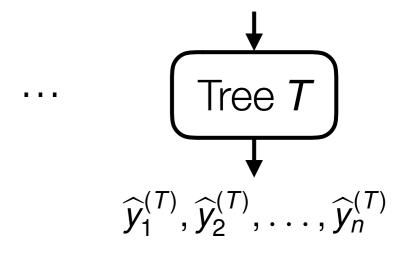
Predicted:
$$\widehat{y}_{1}^{(1)}, \widehat{y}_{2}^{(1)}, \dots, \widehat{y}_{n}^{(1)}$$
 $\widehat{y}_{1}^{(2)}, \widehat{y}_{2}^{(2)}, \dots, \widehat{y}_{n}^{(2)}$

Actual:
$$y_1, y_2, ..., y_n$$
 $y_1, y_2, ..., y_n$



$$\widehat{y}_1^{(2)}, \widehat{y}_2^{(2)}, \ldots, \widehat{y}_n^{(2)}$$

$$y_1, y_2, \ldots, y_n$$



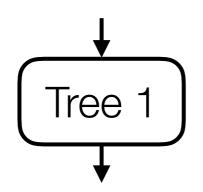
 y_1, y_2, \ldots, y_n

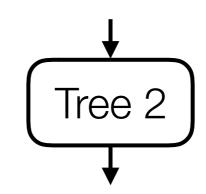
Adjust for how much each tree's votes count

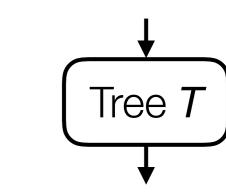
Learn trees sequentially accounting for mistakes made previously

Training data: x_1, x_2, \ldots, x_n x_1, x_2, \ldots, x_n

 X_1, X_2, \ldots, X_n Weights: $w_1^{(1)}, w_2^{(1)}, \dots, w_n^{(1)}, w_1^{(2)}, w_2^{(2)}, \dots, w_n^{(2)}, \dots, w_n^{(2)}, \dots, w_n^{(T)}$







Predicted: $\widehat{y}_1^{(1)}, \widehat{y}_2^{(1)}, \dots, \widehat{y}_n^{(1)}$ $\widehat{y}_1^{(2)}, \widehat{y}_2^{(2)}, \dots, \widehat{y}_n^{(2)}$

Actual: $y_1, y_2, ..., y_n$ $y_1, y_2, ..., y_n$

$$\widehat{y}_1^{(T)}, \widehat{y}_2^{(T)}, \ldots, \widehat{y}_n^{(T)}$$

 y_1, y_2, \ldots, y_n

Adjust for how much each tree's votes count

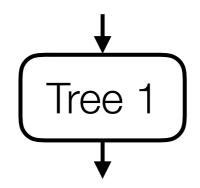
$$similarity(x, x_i) = \sum_{t=1}^{T} \alpha_t similarity_t(x, x_i)$$

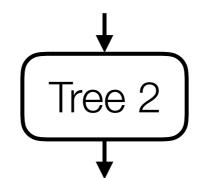
Learn trees sequentially accounting for mistakes made previously

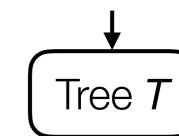
Training data: x_1, x_2, \ldots, x_n x_1, x_2, \ldots, x_n

Weights: $w_1^{(1)}, w_2^{(1)}, \dots, w_n^{(1)}, w_1^{(2)}, w_2^{(2)}, \dots, w_n^{(2)}, \dots, w_n^{(2)}, \dots, w_n^{(T)}, w_2^{(T)}, \dots, w_n^{(T)}$

 X_1, X_2, \ldots, X_n







Predicted: $\widehat{y}_1^{(1)}, \widehat{y}_2^{(1)}, \dots, \widehat{y}_n^{(1)}$ $\widehat{y}_1^{(2)}, \widehat{y}_2^{(2)}, \dots, \widehat{y}_n^{(2)}$

Actual: $y_1, y_2, ..., y_n$ $y_1, y_2, ..., y_n$

$$\widehat{y}_1^{(T)}, \widehat{y}_2^{(T)}, \ldots, \widehat{y}_n^{(T)}$$

 y_1, y_2, \ldots, y_n

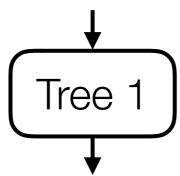
Adjust for how much each tree's votes count

similarity
$$(x, x_i) = \sum_{t=1}^{T} \alpha_t$$
 similarity $_t(x, x_i)$ weight for tree t

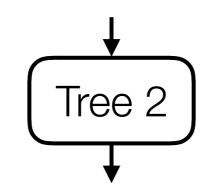
Learn trees sequentially accounting for mistakes made previously

Training data: x_1, x_2, \ldots, x_n x_1, x_2, \ldots, x_n

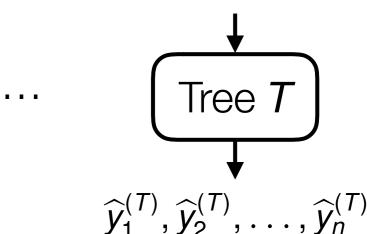
 X_1, X_2, \ldots, X_n Weights: $w_1^{(1)}, w_2^{(1)}, \dots, w_n^{(1)}, w_1^{(2)}, w_2^{(2)}, \dots, w_n^{(2)}, \dots, w_n^{(2)}, \dots, w_n^{(T)}, w_2^{(T)}, \dots, w_n^{(T)}$



Actual: $y_1, y_2, ..., y_n$ $y_1, y_2, ..., y_n$



Predicted: $\widehat{y}_1^{(1)}, \widehat{y}_2^{(1)}, \dots, \widehat{y}_n^{(1)}$ $\widehat{y}_1^{(2)}, \widehat{y}_2^{(2)}, \dots, \widehat{y}_n^{(2)}$



 y_1, y_2, \ldots, y_n

Adjust for how much each tree's votes count

 $similarity(x, x_i) = \sum_{i} \alpha_t similarity_t(x, x_i)$ weight for tree t

Still an adaptive NN method!

Learn trees sequentially accounting for mistakes made previously

Training data: X_1, X_2, \ldots, X_n X_1, X_2, \ldots, X_n

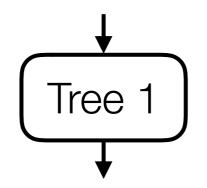
$$X_1, X_2, \ldots, X_n$$

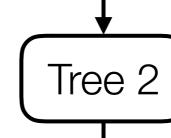
$$X_1, X_2, \ldots, X_n$$

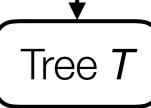
$$W_1^{(1)}, W_2^{(1)}, \ldots, W_n^{(1)}$$

$$W_1^{(2)}, W_2^{(2)}, \ldots, W_n^{(2)}$$

Weights:
$$(w_1^{(1)}, w_2^{(1)}, \dots, w_n^{(1)}, w_1^{(2)}, w_2^{(2)}, \dots, w_n^{(2)}, \dots, w_n^{(2)}, \dots, w_n^{(T)}, w_2^{(T)}, \dots, w_n^{(T)})$$







$$\widehat{y}_1^{(1)}, \widehat{y}_2^{(1)}, \ldots, \widehat{y}_n^{(1)}$$

Predicted:
$$\widehat{y}_1^{(1)}, \widehat{y}_2^{(1)}, \dots, \widehat{y}_n^{(1)} \qquad \widehat{y}_1^{(2)}, \widehat{y}_2^{(2)}, \dots, \widehat{y}_n^{(2)}$$

Actual:

$$y_1, y_2, \ldots, y_n$$

$$y_1, y_2, \ldots, y_n$$

$$\widehat{y}_1^{(T)}, \widehat{y}_2^{(T)}, \ldots, \widehat{y}_n^{(T)}$$

$$y_1, y_2, \ldots, y_n$$

Adjust for how much each tree's votes count

similarity(x, x_i) = $\sum_{i} \alpha_t$ similarity_t(x, x_i)

weight for tree t

Still an adaptive NN method!

Different ways to choose weights yield different boosting methods (e.g., AdaBoost, gradient tree boosting)