# Adaptive Nearest Neighbor Classification and Regression Based on Decision Trees 

slides by<br>George Chen<br>Carnegie Mellon University<br>Fall 2017

## NN and Kernel Classification and Regression



News Activity for \#Barclays

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GMT Time (June 27, 2012)

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How we did this: weighted majority voting
Chen, Nikolov, and Shah. A Latent Source Model for Nonparametric Time Series Classification. NIPS 2013.

Weighted Majority Voting

## Weighted Majority Voting

Test data


## Weighted Majority Voting



Red $=$ viral
Blue $=$ not viral


## Weighted Majority Voting



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## Nearest Neighbor Classification



NN Classification Variants

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- Fixed-radius near neighbor classification: consider all training data at least some similarity threshold close to test data point (i.e., use all training data distance $\leq h$ away)
- Once again, can use weighted or unweighted votes


## Regression: Each label is continuous instead of discrete

## Kernel Regression



## Kernel Regression



## Kernel Regression

Weighted average instead of weighted majority vote


## NN Regression



## NN Regression



Just like classification: $k$-NN and fixed-radius NN variants, also weighted and unweighted
"Adaptive" nearest neighbors: learn the similarity function

## Decision Trees

## Example Made-Up Data



## Example Decision Tree



## Learning a Decision Tree

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- I'll show one way (that nobody actually uses in practice) but it's easy to explain


## Learning a Decision Tree



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1. Pick a random feature


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Within each side, recurse until a


Example termination criteria: $\geq 90 \%$ points within region has same label, number of points within region is $<5$

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For a new person with feature vector (age, weight), easy to predict!

## Nearest Neighbor Interpretation



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## Decision Tree Learned



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Feature space sliced up into leaf cells


## Decision Tree



Feature space sliced up into leaf cells

## Decision Tree

Red: diabetic
Blue: not diabetic

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Red: diabetic
Blue: not diabetic 9 nearest neighbors

Similarity to points in same leaf cell: 1/(\# training points in leaf cell) Similarity to points in other leaf cells: 0


Prediction for test point: majority vote of training points in same leaf cell (these training points act as nearest neighbors to the test point!)

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## Election results

Diabetic: 8/9 votes (winner) Not diabetic: 1/9 votes

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Weight (Ib)

Not diabetic: $1 / 9$ votes

## Election results

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Test point


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Similarity to points in same leaf cell: 1/(\# training points in leaf cell) Similarity to points in other leaf cells: 0

Weighted majority voting using this definition of similarity precisely gives the prediction for this particular decision tree!
$20 \quad 30 \quad 40 \quad 50 \quad$ Age (years)

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- Learning a decision tree learns a similarity function (that depends on labels)


# Decision Tree for Classification Regression 

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Learn each tree
separately using
same training data


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\text { Tree } 2
$$

$$
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$$

$$
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Final prediction: majority vote of the different trees' predictions This is not the only way to aggregate predictions!

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diabetic 8/9 votes
1/4 votes
5/7 votes


2/3 votes not diabetic $1 / 9$ votes $3 / 4$ votes $2 / 7$ votes

1/3 votes

## Decision Forest for Classification

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diabetic $8 / 9$ votes $1 / 4$ votes $\quad 5 / 7$ votes $\quad 2 / 3$ votes


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Nearest neighbor interpretation:
For a specific test data point $x$ and training data point $x_{i}$

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For a specific test data point $x$ and training data point $x_{i}$

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\operatorname{similarity}\left(x, x_{i}\right)=\frac{1}{T} \sum_{t=1}^{T} \operatorname{similarity}_{t}\left(x, x_{i}\right)
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Learn each tree
separately using same training data Tree 1
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## Decision Forest for Classification

Learn each tree Regression


Average these values to get final prediction
Nearest neighbor interpretation:
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similarity $\left(x, x_{i}\right)=\frac{1}{T} \sum_{t=1}^{T} \operatorname{similarity}_{t}\left(x, x_{i}\right){ }^{2}$ between 0 and 1

## Decision Forest



## Decision Forest

Learn each tree
separately using same training data

Combine values to get final prediction
Question: What happens if all the trees are the same?

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$n$ training $\quad$ Decision Forest data points


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Adding randomness can make trees more different!

- Random Forest: in addition to randomly choosing features to threshold, also randomize training data used for each tree
- Extremely randomized trees: further randomize thresholds rather than trying to pick clever thresholds


## Boosting

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I'll only sketch the general idea

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Random decision forests learned each tree separately

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If some trees are bad, we still weight them equally

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Boosting: learn trees sequentially, and learn from previous trees' mistakes

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Random decision forests learned each tree separately

Boosting: learn trees sequentially, and learn from previous trees' mistakes

If some trees are bad, we still weight them equally

Boosting: weight trees unequally so bad trees are down-weighted

## Boosting

## Boosting

Tree 1

## Boosting

Training data


Tree 1

## Boosting

Training data


## Boosting

Training data


## Boosting



Predicted: cat, dog, shark

## Boosting



Predicted: cat, dog, shark
Actual: cat, cat, robot

## Boosting



Predicted: cat, dog, shark
Actual: cat, cat, robot
Where did the errors appear?

## Boosting



Predicted: cat,dog shark
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Where did the errors appear?

## Boosting



Predicted: cat, dog shark
Actual: cat,"cat, wobot
Where did the errors appear?
Duplicate these training examples
to emphasize them more when
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## Boosting



Training data


Predicted: cat, dog shark
Actual: cat, ${ }^{\prime}$ cat, xobot
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Training data


Predicted: cat,'dog shark
Actual: cat, ${ }^{\prime}$ cat, , robot
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## Boosting



Predicted: cat,'dog shark Actual: cat, cat, robot


Predicted: cat, cat, donkey
Actual: cat, cat, robot
Where did the errors appear?
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## Boosting



Predicted: cat,'dog shark Actual: cat, cat, robot

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Predicted: cat,'dog shark Actual: cat, ${ }^{\prime}$ cat, ,

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Predicted: cat, cat,'donkey
Actual: cat, cat,'robot
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## Boosting

Training data: $\quad x_{1}, x_{2}, \ldots, x_{n}$ $x_{1}, x_{2}, \ldots, x_{n}$
Weights: $w_{1}^{(1)}, w_{2}^{(1)}, \ldots, w_{n}^{(1)}$

$w_{1}^{(2)}, w_{2}^{(2)}, \ldots, w_{n}^{(2)}$


Predicted: $\widehat{y}_{1}^{(1)}, \widehat{y}_{2}^{(1)}, \ldots, \widehat{y}_{n}^{(1)}$
Actual: $\quad y_{1}, y_{2}, \ldots, y_{n}$
$y_{1}, y_{2}, \ldots, y_{n}$

$$
x_{1}, x_{2}, \ldots, x_{n}
$$

$$
w_{1}^{(T)}, w_{2}^{(T)}, \ldots, w_{n}^{(T)}
$$


$\widehat{y}_{1}^{(T)}, \widehat{y}_{2}^{(T)}, \ldots, \widehat{y}_{n}^{(T)}$
$y_{1}, y_{2}, \ldots, y_{n}$

## Boosting

Learn trees sequentially accounting for mistakes made previously
Training data: $x_{1}, x_{2}, \ldots, x_{n}$

$$
x_{1}, x_{2}, \ldots, x_{n}
$$

$$
x_{1}, x_{2}, \ldots, x_{n}
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Weights: $w_{1}^{(1)}, w_{2}^{(1)}, \ldots, w_{n}^{(1)} \quad w_{1}^{(2)}, w_{2}^{(2)}, \ldots, w_{n}^{(2)}$


Predicted: $\widehat{y}_{1}^{(1)}, \widehat{y}_{2}^{(1)}, \ldots, \widehat{y}_{n}^{(1)}$

$$
\widehat{y}_{1}^{(2)}, \widehat{y}_{2}^{(2)}, \ldots, \widehat{y}_{n}^{(2)}
$$

Actual: $\quad y_{1}, y_{2}, \ldots, y_{n}$
$y_{1}, y_{2}, \ldots, y_{n}$
$w_{1}^{(T)}, w_{2}^{(T)}, \ldots, w_{n}^{(T)}$


$$
\widehat{y}_{1}^{(T)}, \widehat{y}_{2}^{(T)}, \ldots, \widehat{y}_{n}^{(T)}
$$

$$
y_{1}, y_{2}, \ldots, y_{n}
$$

## Boosting

Learn trees sequentially accounting for mistakes made previously
Training data: $x_{1}, x_{2}, \ldots, x_{n}$

$$
x_{1}, x_{2}, \ldots, x_{n}
$$

$$
x_{1}, x_{2}, \ldots, x_{n}
$$

Weights: $w_{1}^{(1)}, w_{2}^{(1)}, \ldots, w_{n}^{(1)} \quad w_{1}^{(2)}, w_{2}^{(2)}, \ldots, w_{n}^{(2)}$


Predicted: $\widehat{y}_{1}^{(1)}, \widehat{y}_{2}^{(1)}, \ldots, \widehat{y}_{n}^{(1)}$

$$
\widehat{y}_{1}^{(2)}, \widehat{y}_{2}^{(2)}, \ldots, \widehat{y}_{n}^{(2)}
$$

Actual: $\quad y_{1}, y_{2}, \ldots, y_{n}$
$y_{1}, y_{2}, \ldots, y_{n}$
$w_{1}^{(T)}, w_{2}^{(T)}, \ldots, w_{n}^{(T)}$


$$
\begin{gathered}
\widehat{y}_{1}^{(T)}, \widehat{y}_{2}^{(T)}, \ldots, \widehat{y}_{n}^{(T)} \\
y_{1}, y_{2}, \ldots, y_{n}
\end{gathered}
$$

Adjust for how much each tree's votes count

## Boosting

Learn trees sequentially accounting for mistakes made previously
Training data: $x_{1}, x_{2}, \ldots, x_{n}$

$$
x_{1}, x_{2}, \ldots, x_{n}
$$

$$
x_{1}, x_{2}, \ldots, x_{n}
$$

Weights: $w_{1}^{(1)}, w_{2}^{(1)}, \ldots, w_{n}^{(1)} \quad w_{1}^{(2)}, w_{2}^{(2)}, \ldots, w_{n}^{(2)}$


Predicted: $\widehat{y}_{1}^{(1)}, \widehat{y}_{2}^{(1)}, \ldots, \widehat{y}_{n}^{(1)}$

$$
\widehat{y}_{1}^{(2)}, \widehat{y}_{2}^{(2)}, \ldots, \widehat{y}_{n}^{(2)}
$$

Actual: $\quad y_{1}, y_{2}, \ldots, y_{n} \quad y_{1}, y_{2}, \ldots, y_{n}$
$w_{1}^{(T)}, w_{2}^{(T)}, \ldots, w_{n}^{(T)}$


$$
\begin{gathered}
\widehat{y}_{1}^{(T)}, \widehat{y}_{2}^{(T)}, \ldots, \widehat{y}_{n}^{(T)} \\
y_{1}, y_{2}, \ldots, y_{n}
\end{gathered}
$$

Adjust for how much each tree's votes count
$\operatorname{similarity}\left(x, x_{i}\right)=\sum_{t=1}^{T} \alpha_{t} \operatorname{similarity}_{t}\left(x, x_{i}\right)$

## Boosting

Learn trees sequentially accounting for mistakes made previously
Training data: $x_{1}, x_{2}, \ldots, x_{n}$

$$
x_{1}, x_{2}, \ldots, x_{n}
$$

$$
x_{1}, x_{2}, \ldots, x_{n}
$$

Weights: $w_{1}^{(1)}, w_{2}^{(1)}, \ldots, w_{n}^{(1)} \quad w_{1}^{(2)}, w_{2}^{(2)}, \ldots, w_{n}^{(2)}$


Predicted: $\widehat{y}_{1}^{(1)}, \widehat{y}_{2}^{(1)}, \ldots, \widehat{y}_{n}^{(1)}$

$$
\widehat{y}_{1}^{(2)}, \widehat{y}_{2}^{(2)}, \ldots, \widehat{y}_{n}^{(2)}
$$

$$
\text { Actual: } \quad y_{1}, y_{2}, \ldots, y_{n} \quad y_{1}, y_{2}, \ldots, y_{n}
$$

$$
w_{1}^{(T)}, w_{2}^{(T)}, \ldots, w_{n}^{(T)}
$$



$$
\begin{gathered}
\widehat{y}_{1}^{(T)}, \widehat{y}_{2}^{(T)}, \ldots, \widehat{y}_{n}^{(T)} \\
y_{1}, y_{2}, \ldots, y_{n}
\end{gathered}
$$

Adjust for how much each tree's votes count

$$
\begin{gathered}
\operatorname{similarity}\left(x, x_{i}\right)=\sum_{t=1}^{T} a_{t} \hat{i}_{t} \operatorname{similarity}{ }_{t}\left(x, x_{i}\right) \\
\text { weight for tree } t
\end{gathered}
$$

## Boosting

Learn trees sequentially accounting for mistakes made previously

Training data: $x_{1}, x_{2}, \ldots, x_{n}$
$x_{1}, x_{2}, \ldots, x_{n}$ $x_{1}, x_{2}, \ldots, x_{n}$
Weights: $w_{1}^{(1)}, w_{2}^{(1)}, \ldots, w_{n}^{(1)} \quad w_{1}^{(2)}, w_{2}^{(2)}, \ldots, w_{n}^{(2)}$

Predicted: $\widehat{y}_{1}^{(1)}, \widehat{y}_{2}^{(1)}, \ldots, \widehat{y}_{n}^{(1)}$
Actual: $\quad y_{1}, y_{2}, \ldots, y_{n}$

$\widehat{y}_{1}^{(2)}, \widehat{y}_{2}^{(2)}, \ldots, \widehat{y}_{n}^{(2)}$
$y_{1}, y_{2}, \ldots, y_{n}$
$w_{1}^{(T)}, w_{2}^{(T)}, \ldots, w_{n}^{(T)}$


$$
\widehat{y}_{1}^{(T)}, \widehat{y}_{2}^{(T)}, \ldots, \widehat{y}_{n}^{(T)}
$$

$$
y_{1}, y_{2}, \ldots, y_{n}
$$

Adjust for how much each tree's votes count
$\operatorname{similarity}\left(x, x_{i}\right)=\sum_{t=1}^{T} \underbrace{}_{t} \operatorname{cosimilarity}_{t}\left(x, x_{i}\right)$
weight for tree $t$
Still an adaptive NN method!

## Boosting

Learn trees sequentially accounting for mistakes made previously
Training data:

$$
x_{1}, x_{2}, \ldots, x_{n}
$$

$$
x_{1}, x_{2}, \ldots, x_{n}
$$

Weights: $w_{1}^{(1)}, w_{2}^{(1)}, \ldots, w_{n}^{(1)} \quad w_{1}^{(2)}, w_{2}^{(2)}, \ldots, w_{n}^{(2)} \quad-----w_{1}^{(T)}, w_{2}^{(T)}, \ldots, w_{n}^{(T)}$


Predicted: $\widehat{y}_{1}^{(1)}, \widehat{y}_{2}^{(1)}, \ldots, \widehat{y}_{n}^{(1)}$
Actual: $\quad y_{1}, y_{2}, \ldots, y_{n}$

$\widehat{y}_{1}^{(2)}, \widehat{y}_{2}^{(2)}, \ldots, \widehat{y}_{n}^{(2)}$
$y_{1}, y_{2}, \ldots, y_{n}$

$\widehat{y}_{1}^{(T)}, \widehat{y}_{2}^{(T)}, \ldots, \widehat{y}_{n}^{(T)}$ $y_{1}, y_{2}, \ldots, y_{n}$

Adjust for how much each tree's votes count

$$
\begin{array}{cc}
\operatorname{similarity}\left(x, x_{i}\right)=\sum_{t=1}^{T} \alpha_{t} \operatorname{sinmilarity}_{t}\left(x, x_{i}\right) & \begin{array}{c}
\text { Different ways to choose } \\
\text { weights yield different } \\
\text { boosting methods }
\end{array} \\
\text { weight for tree } t & \text { (e.g., AdaBoost, gradient }
\end{array}
$$

Still an adaptive NN method!

